MATH 122

VOLUMES BY SLICING.

GOALS

- ☐ KNOW GENERAL INTEGRAL FORMULA FOR VOLUMES BY SLICING
- ☐ COMPUTE VOLUMES FOR SOLIDS OF REVOLUTION
  - ☐ REVOLVE ABOUT BOTH AXES AND OTHER LINES
- ☐ COMPUTE VOLUMES FOR SOLIDS WITH CROSS-SECTIONS LIKE SQUARES, SEMICIRCLES, AND TRIANGLES

MAIN IDEA: IF A SOLID HAS CROSS-SECTIONS WITH AREA $A(x)$, THEN THE VOLUME IS

$$\int_a^b A(x)\,dx$$

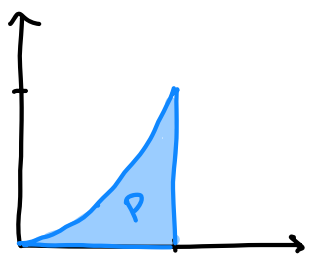
$A(x)$: AREA OF CROSS-SECTION; dx: THICKNESS OF "SLICE"

SOLIDS OF REVOLUTION: SOLIDS CREATED BY REVOLVING A 2D REGION ABOUT AN AXIS.

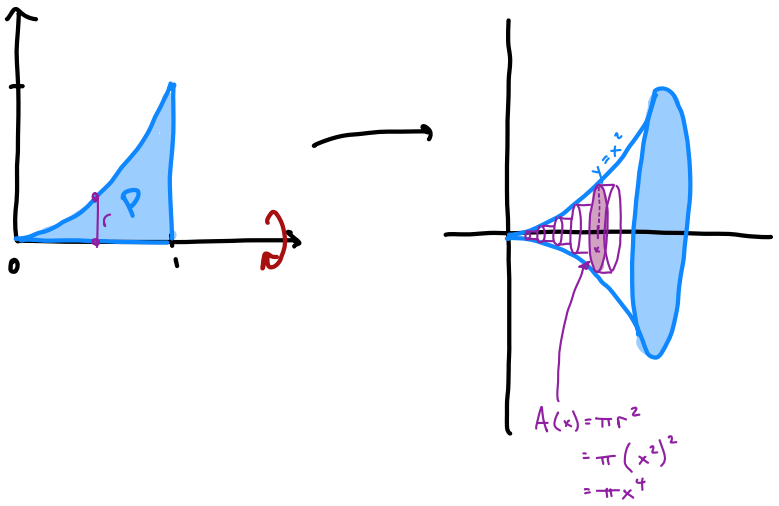
$$A(x) = \pi r^2 \quad \text{OR} \quad A(x) = \pi R^2 - \pi r^2$$

* R AND r ARE FUNCTIONS OF x *

Ex: P: REGION ENCLOSED BY $y = x^2$, $y = 0$, $x = 1$



REVOLVE P ABOUT x-AXIS



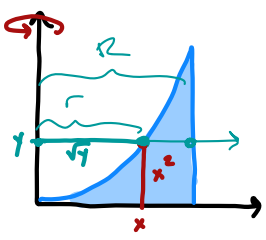
$$V=\int_0^1 \pi x^4\,dx$$

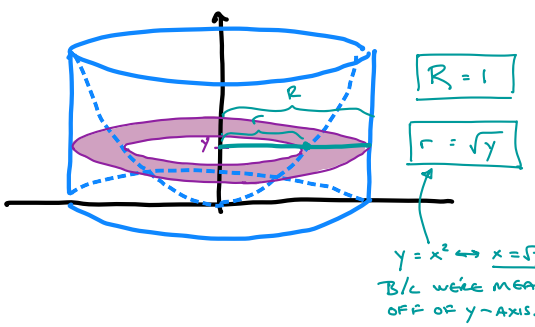
$$=\frac{\pi}{5}x^5\Big|_0^1$$

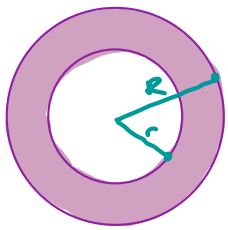
$$=\boxed{\frac{\pi}{5}}$$

REVOLVE P ABOUT y-AXIS

NOT $\pi(R-r)^2$!! ☹







$A=\pi R^2-\pi r^2$

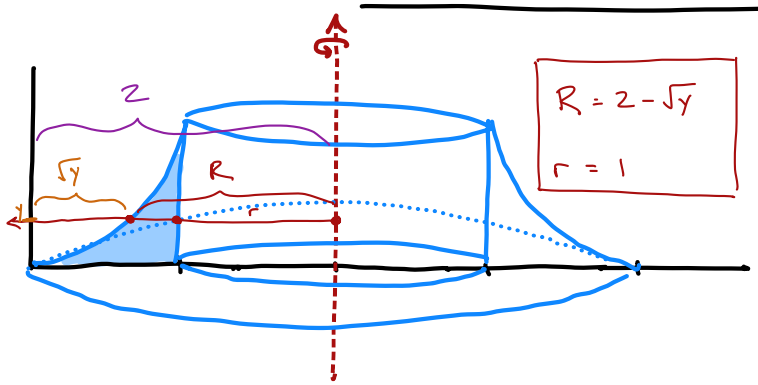
$$V=\int_0^1 \pi(1)^2-\pi(\sqrt{y})^2\,dy$$

$$=\pi\int_0^1 1-y\,dy$$

$$=\pi\left(y-\frac{1}{2}y^2\Big|_0^1\right)$$

$$=\boxed{\frac{\pi}{2}}$$

REVOLVE ABOUT x = 2

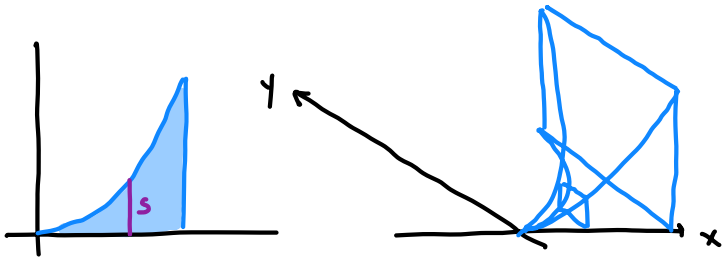


$R=2-\sqrt{y}$

$r=1$

$$V=\int_0^1 \pi(2-\sqrt{y})^2-\pi(1)^2\,dy$$

CROSS-SECTIONS PERPENDICULAR TO x-AXIS ARE SQUARES

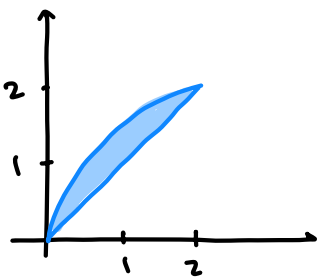


$s = x^2$

$A(x) = s^2 = x^4$

$$V = \int_0^1 x^4 \, dx = \cdots = \boxed{\frac{1}{5}}$$

Ex: $y = \sqrt{2x}$; $y = x$



1) REVOLVE ABOUT x-AXIS

2) REVOLVE ABOUT y-AXIS

3) REVOLVE ABOUT $x = -1$

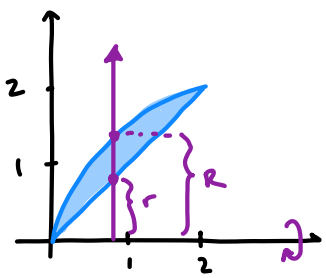
4) REVOLVE ABOUT $y = 3$

5) CROSS-SEC. PERP. TO y-AXIS ARE SQUARES

6) CROSS-SEC. PERP TO y-AXIS ARE SEMICIRCLES.

*BOUNDS COME FROM REGION (2D) NOT SOLID (3D).

1) REVOLVE ABOUT x-AXIS



$r = x$

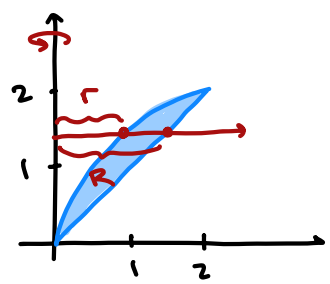
$R = \sqrt{2x}$

DRAW LINE PERP. TO AXIS OF REV. THROUGH REGION

- FIRST LINE WE HIT IS r
- SECOND LINE WE HIT IS R

$$V = \pi \int_0^2 \left(\sqrt{2x}\right)^2 - (x)^2 \, dx$$

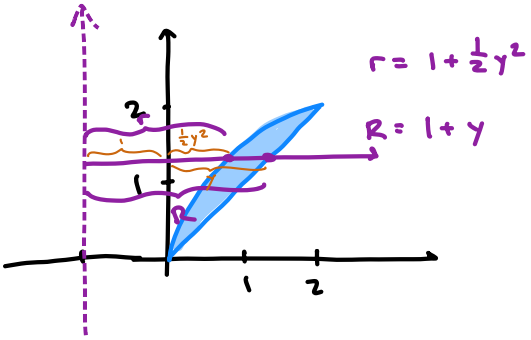
2) REVOLVE ABOUT y-AXIS



$r = \frac{1}{2}y^2$ $R = y$

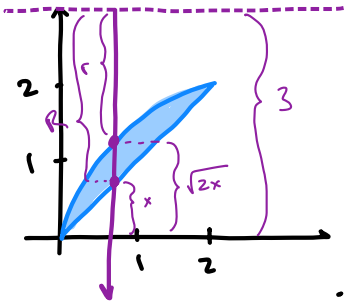
* MEASURING FROM y-AXIS, IT'S A dy INTEGRAL *

$$\pi \int_0^2 (y)^2 - \left(\tfrac{1}{2}y^2\right)^2 \, dy$$



3) REVOLVE ABOUT $x = -1$

$$\pi \int_0^2 (1+y)^2 - \left(1 + \tfrac{1}{2}y^2\right)^2 dy$$

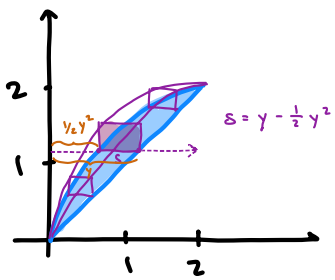


4) REVOLVE ABOUT $y = 3$

$$\pi \int_0^2 (3-x)^2 - \left(3 - \sqrt{2x}\right)^2 dx$$

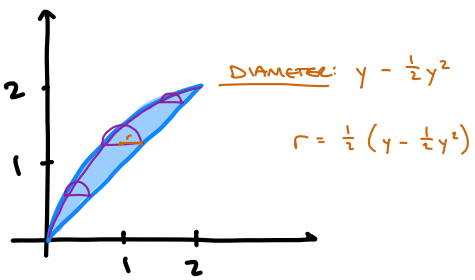
$r = 3 - \sqrt{2x}$

$R = 3 - x$



5) CROSS-SEC. PERP. TO Y-AXIS ARE SQUARES

$$\int_0^2 \left(y - \tfrac{1}{2}y^2\right)^2 dy$$



6) CROSS-SEC. PERP TO Y-AXIS ARE SEMICIRCLES.

$$\boxed{\int_0^2 \tfrac{1}{2}\pi \left(\tfrac{1}{2}\left(y - \tfrac{1}{2}y^2\right)\right)^2 dy}$$