MATH 122

PARAMETRIC CURVES

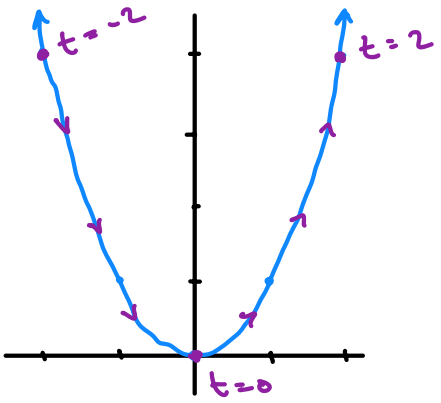
GOALS

- ☐ IDENTIFY CURVES AND THEIR ORIENTATION FROM PARAMETRIC EQUATIONS
- ☐ ELIMINATE THE PARAMETER
- ☐ COMPUTE $\frac{dy}{dx}$ AND $\frac{d^2y}{dx^2}$ FROM PARAMETRIC EQUATIONS
- ☐ FIND AN EQUATION FOR THE LINE TANGENT TO A PARAMETRIC CURVE AT $t = t_0$

e.g.

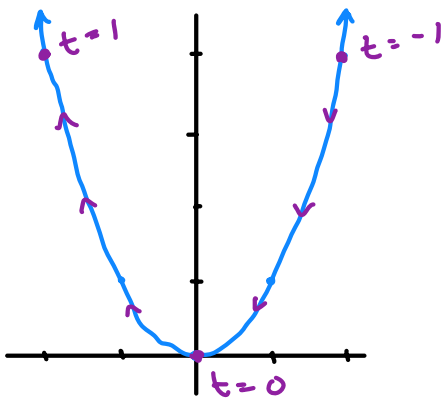
$$\begin{cases} x = t \\ y = t^2 \end{cases}$$

t	(x,y)
-2	$(-2,4)$
0	$(0,0)$
2	$(2,4)$



$$\begin{cases} x = -2t \\ y = 4t^2 \end{cases}$$

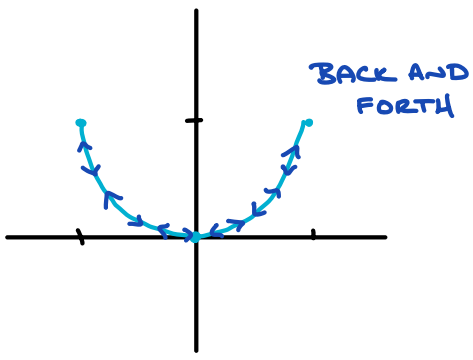
t	(x,y)
1	$(-2,4)$
0	$(0,0)$
-1	$(2,4)$



BOTH HAVE SHAPE $y = x^2$, BUT DIFFERENT ORIENTATIONS AND SPEEDS

BONUS:

$$\begin{cases} x = \sin t \\ y = \sin^2 t \end{cases}$$



Ex:

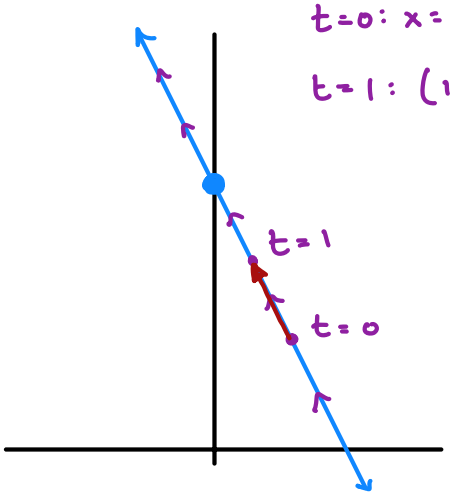
$$\begin{cases} x = 2 - t \\ y = 3 + 2t \end{cases}$$

ELIMINATE THE PARAMETER:

$x = 2 - t \longrightarrow t = 2 - x$

$y = 3 + 2(2 - x)$

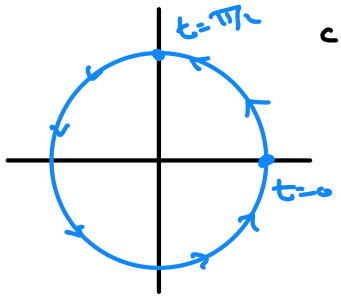
$\boxed{y = -2x + 7}$



$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2}{-1} = \boxed{-2}$$

Ex:

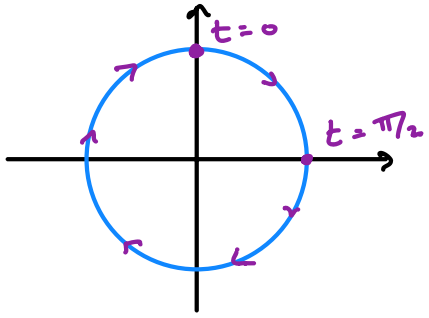
$$\begin{cases} x = \cos t \\ y = \sin t \end{cases}$$



$\cos^2 t + \sin^2 t = 1$

$\boxed{x^2 + y^2 = 1}$

$$\begin{cases} x = \sin t \\ y = \cos t \end{cases}$$



$x^2 + y^2 = 1$

$$\begin{cases} x = 3\cos t \\ y = 2\sin t \end{cases}$$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

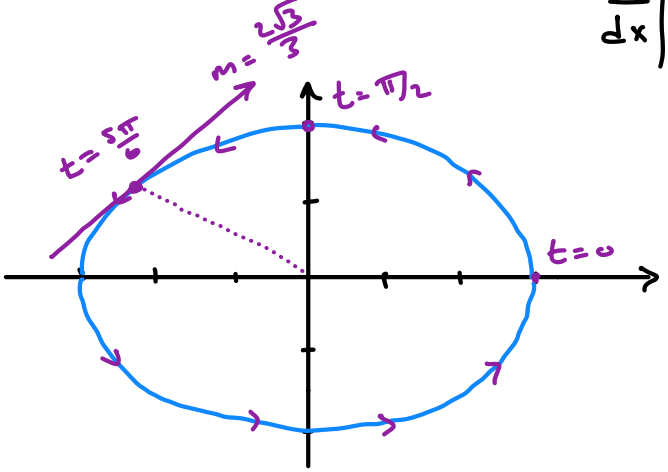
$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$$\begin{cases} x = 3\cos t \\ y = 2\sin t \end{cases}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{-3\sin t} = \boxed{-\frac{2}{3}\cot t}$$

GIVES SLOPE OF TANGENT:

$$\left.\frac{dy}{dx}\right|_{t=\frac{5\pi}{6}} = -\frac{2}{3}\left(-\sqrt{3}\right) = \frac{2\sqrt{3}}{3}$$



HOW ABOUT $\frac{d^2y}{dx^2}$?

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy'}{dx} = \frac{dy'/dt}{dx/dt} = \frac{\frac{2}{3}\csc^2 t}{-3\sin t}$$

$$= \boxed{-\frac{2}{9}\csc^3 t}$$

command: $\frac{d}{dx}$; y': $\frac{dy}{dx}$

Ex: $\begin{cases} x = \sin t \\ y = 4\cos t \end{cases}$

ARC LENGTH: $$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$$

$x' = e^t \cos t - e^t \sin t \longrightarrow (x')^2 = e^{2t}\cos^2 t - 2e^{2t}\cos t \sin t + e^{2t}\sin^2 t$

$y' = e^t \sin t + e^t \cos t \longrightarrow (y')^2 = e^{2t}\sin^2 t + 2e^{2t}\cos t \sin t + e^{2t}\cos^2 t$

$$(x')^2 + (y')^2 = 2e^{2t}\cos^2 t + 2e^{2t}\sin^2 t$$

$$= 2e^{2t}\underbrace{(\cos^2 t + \sin^2 t)}_{1}$$

$$L = \int_0^{\pi} \sqrt{2e^{2t}}\, dt = \int_0^{\pi} \sqrt{2}\, e^t\, dt = \sqrt{2}\, e^t \Big|_0^{\pi}$$

$$= \boxed{\sqrt{2}\, e^{\pi} - \sqrt{2}}$$