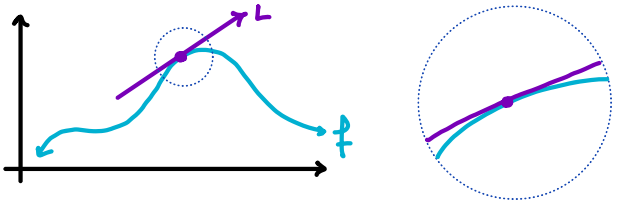
MATH 121

LOCAL LINEAR APPROXIMATION

GOALS

- ☐ KNOW HOW TO COMPUTE THE LOCAL LINEAR APPROXIMATION (LINEARIZATION) OF $f(x)$ AT $x = a$
- ☐ APPLY LOCAL LINEAR APPROXIMATION

MAIN IDEA FOR A DIFFERENTIABLE (CONTINUOUS AND SMOOTH) FUNCTION, WE CAN USE THE TANGENT LINE AT $x = x_0$ TO APPROXIMATE THE FUNCTION NEARBY.



e.g. $\sqrt[3]{7.8}$

- DEFINE f: $f(x) = \sqrt[3]{x}$
- CHOOSE PT. OF TANGENCY CLOSE TO 7.8 WHERE WE CAN EVALUATE f.

$$x_0 = 8$$

- FIND TANGENT LINE:

$$y_0 = f(8) = \sqrt[3]{8} = 2$$

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$$

$$m = f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12}$$

$$y - 2 = \frac{1}{12}(x - 8)$$

$$\boxed{y = \frac{1}{12}(x-8) + 2}$$ ← OFTEN CALLED $L(x)$ INSTEAD OF y.

$$\boxed{L(x) = \frac{1}{12}(x-8) + 2}$$

· USE $L(x)$ TO APPROXIMATE f:

$$\sqrt[3]{7.8} \approx \frac{1}{12}(7.8 - 8) + 2 \qquad \text{PLUG 7.8 INTO TANGENT LINE.}$$

$$\approx \frac{1}{12}(-0.2) + 2$$

$$\approx \frac{-1}{60} + 2$$

$$\approx \boxed{\frac{119}{60}}$$

CALCULATOR:

$\sqrt[3]{7.8} = 1.983179\ldots$

$\frac{119}{60} = 1.983333\ldots$

FORMULA:

$$y - y_0 = m(x - x_0)$$

$$y = m(x - x_0) + y_0$$

$$\boxed{L(x) = f'(x_0)(x - x_0) + f(x_0)}$$

EX: FIND LINEARIZATION OF $f(x) = e^x$ AT $x = 0$. LINEARIZATION = TANGENT LINE.

$L(x) = Ax + B$

* WE WANT $f(0) = L(0)$ AND $f'(0) = L'(0)$

$f'(x) = e^x$ $\qquad$ $L'(x) = A$

$f(0) = L(0)$ $\qquad$ $f'(0) = L'(0)$

$e^0 = A(0) + B$ $\qquad$ $e^0 = A$

$\boxed{1 = B}$ $\qquad$ $\boxed{1 = A}$

$$\boxed{L(x) = x + 1}$$

Ex: $f(x) = \sin(2x)$ AT $x_0 = 0$

FAMILIAR WAY:

$y_0 = f(0) = 0$

$f'(x) = 2\cos 2x$

$m = f'(0) = 2$

$y - 0 = 2(x - 0)$

$\boxed{y = 2x}$

ALT: $L(x) = Ax + B$

$f(0) = L(0)$

$\boxed{0 = B}$

$f'(0) = L'(0)$

$2\cos 0 = A$

$\boxed{A = 2}$

$\boxed{L(x) = 2x}$

Ex: APPROXIMATE $\arctan(1/4)$ USING L.L.A.

$f(x) = \arctan(x)$; $x_0 = 0$

$y_0 = f(0) = 0$

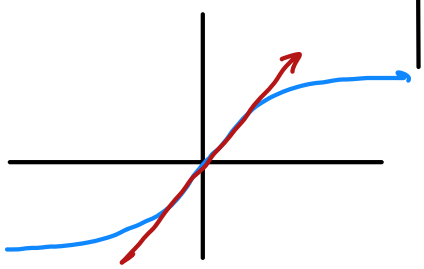
$f'(x) = \frac{1}{1+x^2}$

$m = f'(0) = 1$

$y - 0 = 1(x - 0)$

$y = x$

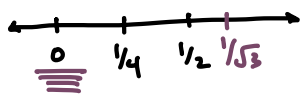
$\arctan(1/4) \approx \boxed{1/4}$



$\arctan(0) = 0$

$\arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

WHICH IS CLOSER TO $1/4$?



$3 < 4 < 16$

$\sqrt{3} < 2 < 4$

$1/4 < 1/2 < 1/\sqrt{3}$

Ex: APPROXIMATE $\sqrt{15.8}$

$f(x) = \sqrt{x}$; $x_0 = 16$

$f(16) = 4$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(16) = \frac{1}{8}$

$L(x) = \frac{1}{8}(x - 16) + 4$

$\sqrt{15.8} \approx L(15.8)$

$\approx \frac{1}{8}(15.8 - 16) + 4$

$\approx \frac{1}{8}(-0.2) + 4$

$\approx 4 - 0.025$

$\approx \boxed{3.975}$