MATH 121

LIMITS OF TRIG FUNCTIONS

GOALS

- ☐ KNOW BASIC LIMITS FOR ALL SIX TRIG FUNCTIONS
- ☐ APPLY THE SQUEEZ THEOREM TO COMPLEX LIMITS
- ☐ DERIVE AND APPLY $\lim_{x \to 0} \frac{\sin x}{x}$
- ☐ USE SUBSTITUTION TO SIMPLIFY COMPLEX LIMITS

BASIC TRIG LIMITS

$\sin x$ & $\cos x$

SINCE $\sin x$ AND $\cos x$ ARE CONTINUOUS EVERYWHERE...

$$\lim_{x \to a} \sin x = \sin a \qquad \lim_{x \to a} \cos x = \cos a.$$

e.g. $\lim_{x \to -\frac{\pi}{3}} \sin x$

BUT, SINCE $\sin x$ AND $\cos x$ BOUNCE BETWEEN -1 AND 1...

THE OTHER FOUR

$\tan x$, $\cot x$, $\sec x$, AND $\csc x$ ALL HAVE VERTICAL ASYMPTOTES SO WE CAN USE OUR APPROACH TO "CASE 3" LIMIT $(K/0)$

EXAMPLES

1. $\lim_{x \to \pi/2^+} \sec x =$

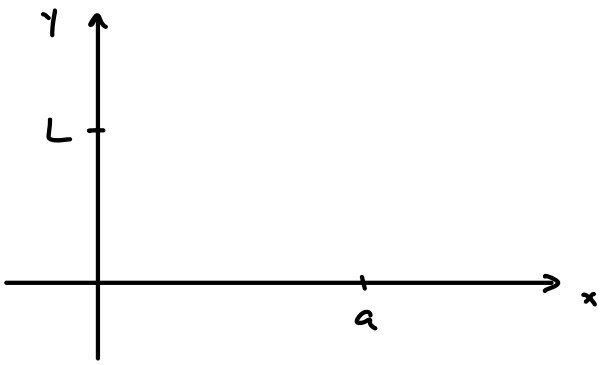
2. $\lim_{x \to 0^-} \cot x$

3. $\lim_{x \to \pi^+} e^{\csc x}$

THE SQUEEZE THEOREM

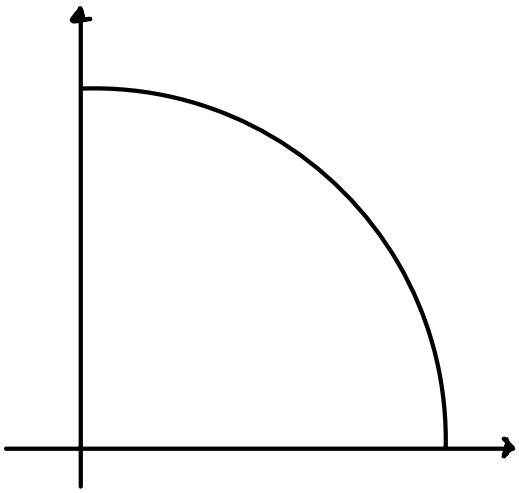
IF $f(x) \leq g(x) \leq h(x)$ AND THEY'RE ALL CONTINUOUS, $(x \neq a)$

IF $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$ THEN $\lim_{x \to a} g(x) = L$



e.g $\lim_{x \to +\infty} \frac{1}{x^2} \cos\left(\frac{1}{x^2}\right)$

DERIVING $\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$:



EXAMPLES

1. $\lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta}$

2. $\lim_{x \to 0} \frac{\sin(5x)}{2x}$

3. $\lim_{x \to 0} \frac{\sin^2(5x)}{2x^2}$

4. $\lim_{x \to 0} \frac{\tan 2x}{\sin 3x}$

5. $\lim_{x \to -\infty} \frac{\sin(e^x)}{e^x}$