MATH 121

LIMITS AT INFINITY

GOALS

- ☐ DEFINE LIMITS AT INFINITY / END BEHAVIOR
- ☐ USE END BEHAVIOR OF $\frac{1}{x}$ TO DERIVE END BEHAVIOR OF POLYNOMIALS AND RATIONAL FUNCTIONS
- ☐ DEFINE HORIZONTAL ASYMPTOTES IN TERMS OF END BEHAVIOR
- ☐ APPLY THE "LEADING TERM TRICK" TO COMPUTE END BEHAVIOR OF POLYNOMIALS, RATIONAL FUNCTIONS, AND OTHER SPECIAL CASES.

END BEHAVIOR OF A FUNCTION

IF WE CAN MAKE $f(x)$ AS CLOSE TO SOME FINITE VALUE L AS WE LIKE BY MAKING x LARGE ENOUGH, FOR $x > 0$, WE WRITE...

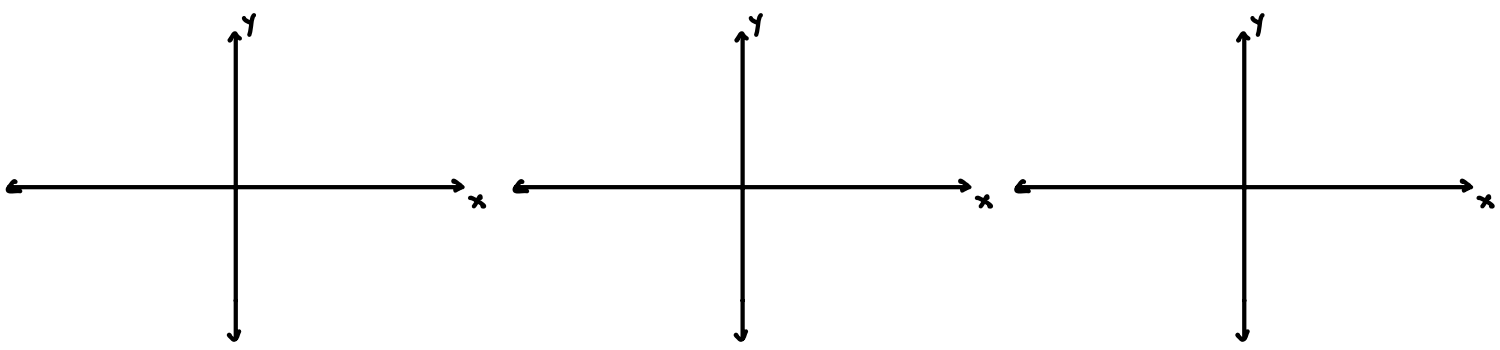
IF WE CAN MAKE $f(x)$ AS CLOSE TO SOME FINITE VALUE L AS WE LIKE BY MAKING x LARGE ENOUGH, FOR $x < 0$, WE WRITE...

* THESE ARE _____. ALSO CALLED THE _____ OF f *

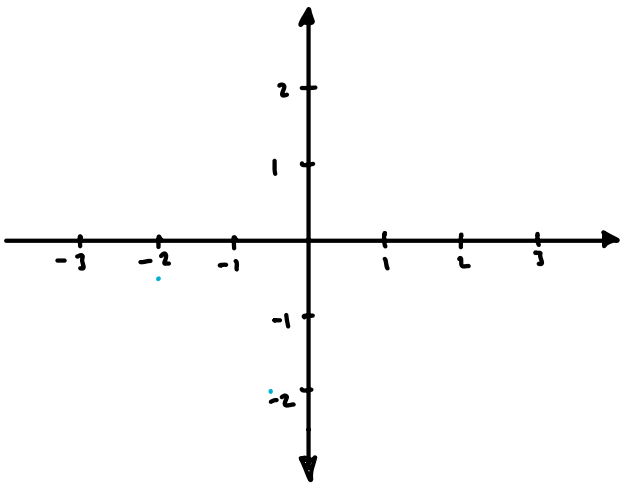
HORIZONTAL ASYMPTOTES

IF $\lim_{x \to +\infty} f(x) = L$ OR $\lim_{x \to -\infty} f(x) = L$, $y = L$ IS A HORIZONTAL ASYMPTOTE.

A FUNCTION CAN HAVE AT MOST TWO HORIZONTAL ASYMPTOTES...



OUR STARTING POINT: $f(x) = \frac{1}{x}$



x	
$f(x)$	

x	
$f(x)$	

ASYMPTOTES:

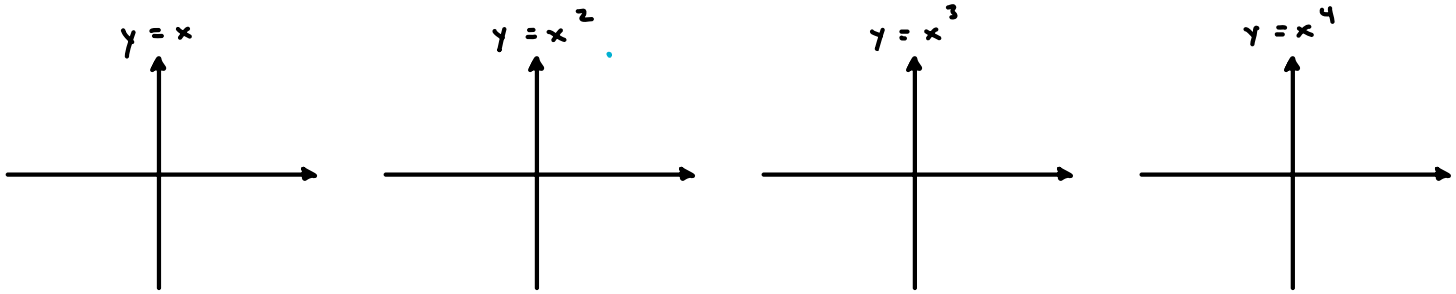
- **HORIZONTAL:**
- **VERTICAL:**

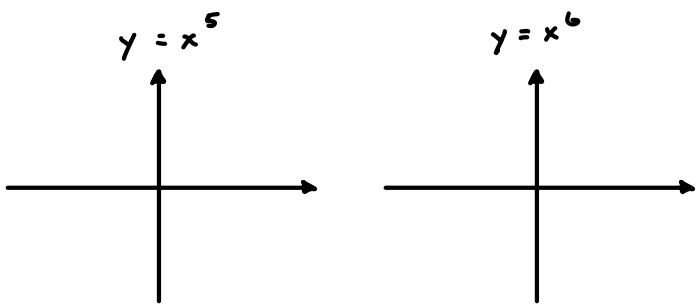
EXTENDING THIS LIMIT: SINCE $\lim_{x \to \infty} 1/x = 0$, IT FOLLOWS

LIMITS OF POWER FUNCTIONS

DEF: A POWER FUNCTION IS A FUNCTION OF THE FORM

LOOKING FOR A PATTERN





$\lim_{x \to +\infty} x^n =$

$\lim_{x \to -\infty} x^n =$

EXAMPLES

1. $\lim_{x \to +\infty} x^{11} =$

2. $\lim_{x \to -\infty} x^{11} =$

3. $\lim_{x \to +\infty} x^{20} =$

4. $\lim_{x \to -\infty} x^{20} =$

LIMITS OF POLYNOMIALS

DEF A POLYNOMIAL IS A FUNCTION OF THE FOLLOWING FORM

$$p(x) =$$

APPLYING WHAT WE KNOW

$$\lim_{x \to \infty} a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

EXAMPLES

1. $\lim_{x \to \infty} 3x^4 - 7x^2 + 5x + 1 =$

2. $\lim_{x \to -\infty} 5 + x - x^3 =$

LIMITS OF RATIONAL FUNCTIONS

DEF A RATIONAL FUNCTION IS

$$r(x) =$$

APPLYING WHAT WE KNOW

$$\lim_{x \to \infty} r(x) =$$

EXAMPLES

1. $\lim_{x \to -\infty} \dfrac{5x^4 + 7x - 1}{3x^5 + 2x^4 + 3} =$

2. $\lim_{x \to +\infty} \dfrac{x^2 + 5x + 6}{4x^2 + x - 1} =$

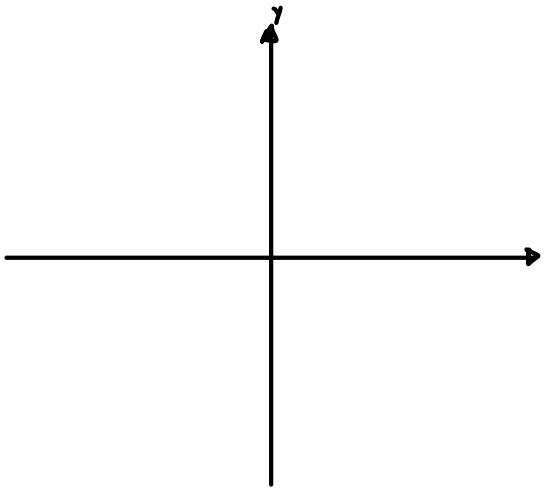
3. $\lim_{x \to -\infty} \dfrac{x^6 - 1}{x^3 + 2x + 5} =$

OTHER CASES

1. $\lim_{x \to -\infty} \dfrac{\sqrt{5x^2 + 1}}{3 - x}$

2. $\lim_{x \to -\infty} \sqrt{x^2 + 3x} + x$

LOGS + EXPONENTIALS



EXAMPLES

1. $\lim_{x \to \infty} \dfrac{3}{e^x + 1}$

2. $\lim_{x \to \infty} \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

3. $\lim_{x \to -\infty} \dfrac{7}{8 + e^x}$

4. $\lim_{x \to 0^+} \dfrac{\ln x + 5}{2 \ln x - 1}$

5. $\lim_{x \to \infty} \frac{\ln 5x^2}{\ln 2x}$

EXTRA EXAMPLES

1. $\lim_{x \to +\infty} \dfrac{5 - x}{x + 7}$

2. $\lim_{x \to -\infty} \dfrac{4x^3 + x^2 + 3x - 1}{7x^3 + 2x^2 - x + 3}$