MATH 121
L'HÔPITAL'S RULE

GOALS

- ☐ KNOW ALL OF THE INDETERMINATE FORMS (FOR WHEN L'HÔPITAL'S RULE APPLIES)
- ☐ USE L'HÔPITALS RULE

MAIN IDEA ALLOWS US TO WORK WITH LIMITS IN INDETERMINATE FORMS.

MAIN FORMS:

SECONDARY FORMS:

L'HÔPITAL FOR $\frac{0}{0}$ SUPPOSE f & g ARE CONTINUOUS AND DIFFERENTIABLE AT $x=a$, AND THAT $\lim_{x\to a} f(x) = 0 = \lim_{x\to a} g(x)$

$$\lim_{x\to 0} \frac{f(x)}{g(x)} =$$

e.g $\lim_{x \to 1} \dfrac{\ln(x)}{x-1}$

L'HÔP WORKS THE EXACT SAME WAY FOR $\frac{\infty}{\infty}$

Ex $\lim_{x \to \infty} \dfrac{x}{e^x}$

Ex e^x WILL ALWAYS BEAT A POWER FUNCTION IN THE RACE TO ∞.

$$\lim_{x \to \infty} \frac{x^{100}}{e^x}$$

TRY

① $\lim_{x \to 0} \dfrac{\tan(3x)}{\ln(x+1)}$

② $\lim_{x\to\infty} \frac{\ln x}{x}$

RECAP

L'HÔPITAL TELLS US THAT $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$ PROVIDED THAT...

TRY:

① $\lim_{x\to 1} \frac{x^9 - 1}{x^5 - 1}$ ② $\lim_{x\to\infty} \frac{\ln(x^2)}{\sqrt{x}}$ ③ $\lim_{x\to 0} \frac{\arctan x}{x}$

④ $\lim_{x\to 1} \frac{1 - x + \ln x}{1 + \cos(\pi x)}$

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OTHER INDET. FORMS: REWRITE AS OR RELATE TO $\frac{0}{0}$ OR $\frac{\infty}{\infty}$.

$0 \cdot \infty$: $\lim_{x \to \infty} x e^{-2x}$

$0 \cdot \infty$ ISN'T ALWAYS 0: $\lim_{x \to 0^+} x \csc x \longrightarrow 0 \cdot \infty$

1^∞:

$$\lim_{x \to 0^+} (1 + 2x)^{5/x}$$