MATH 122

GOALS

- ☐ REWRITE ALL OF OUR FAMILIAR DERIVATIVE RULES AS ANTIDERIVATIVES
- ☐ COMBINE ANTIDERIVATIVE RULES
- ☐ UNDERSTAND AND SOLVE INITIAL VALUE PROBLES

$$\int f(x)dx = F(x) + C$$

* $F(x) + C$ IS THE GENERAL FORM FOR ALL ANTIDERIVATIVES OF $f(x)$ $(F' = f)$ *

POWER RULE:

DERIVATIVE	ANTIDERIVATIVE
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad (n \neq -1)$
• MULT. BY OLD EXPONENT	• ADD 1 TO EXPONENT
• SUBTRACT 1 FROM EXPONENT	• DIVIDE BY NEW EXPONENT

EXAMPLES

① $\int x^4 dx = \boxed{\frac{1}{5} x^5 + C}$

CHECK: $\frac{d}{dx}\left(\frac{1}{5} x^5\right) = \frac{1}{5} \cdot 5x^4 = x^4$ ✓

② $\int \sqrt[3]{x}\, dx = \int x^{1/3} dx$

$= \frac{1}{\frac{4}{3}} x^{4/3} + C$

$= \boxed{\frac{3}{4} x^{4/3} + C}$

ALWAYS REWRITE $\sqrt{\ }$'S AS EXPONENTS

③ $\int \frac{1}{x^5}\,dx = \int x^{-5}\,dx$

$= \frac{1}{-4}x^{-4} + C$

$= \boxed{-\frac{1}{4x^4} + C}$

How about $\frac{1}{x}$?

$$\boxed{\int \frac{1}{x}\,dx = \ln|x| + C}$$

THE DOMAIN $1/x$ IS $x \neq 0$

BUT THE DOMAIN $\ln x$ IS $x > 0$

WHY? B/C $\ln(-2) = ?$ MEANS $e^{?} = -2$ BUT IT CAN'T!

e.g. $\int_1^2 \frac{1}{x}\,dx = \ln|x|\Big|_1^2$

$= \ln 2 - \ln 1$

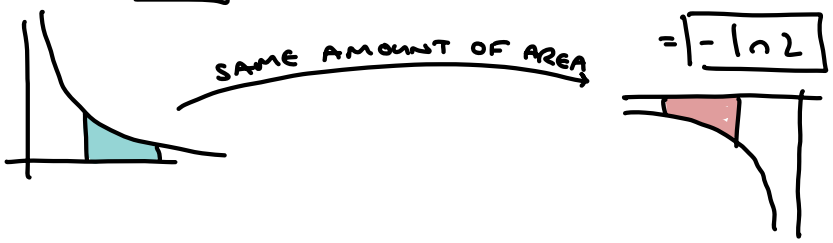
$= \boxed{\ln 2}$

$\int_{-2}^{-1} \frac{1}{x}\,dx = \ln|x|\Big|_{-2}^{-1}$

$= \ln|-1| - \ln|-2|$

$= \ln 1 - \ln 2$

$= \boxed{-\ln 2}$



EXPONENTIALS:

$$\boxed{\int e^x\,dx = e^x + C}$$

SINCE $\frac{d}{dx}(b^x) = \underbrace{(\ln b)}_{\text{CONSTANT}} b^x$, $\boxed{\int b^x\,dx = \frac{1}{\ln b}b^x + C}$

e.g. $\int 5^x\,dx = \frac{1}{\ln 5}\cdot 5^x + C$ | CHECK: $\frac{d}{dx}\left(\frac{1}{\ln 5}5^x\right) = \frac{1}{\ln 5}(\ln 5)\cdot 5^x$ ✓

TRIG:

$\frac{d}{dx}(\sin x) = \cos x \longrightarrow \int \cos x\,dx = \sin x + C$

$\frac{d}{dx}(\cos x) = -\sin x \longrightarrow \int \sin x\,dx = -\cos x + C$

$\frac{d}{dx}(\tan x) = \sec^2 x \longrightarrow \int \sec^2 x\,dx = \tan x + C$

$\frac{d}{dx}(\cot x) = -\csc^2 x \longrightarrow \int \csc^2 x\,dx = -\cot x + C$

$\frac{d}{dx}(\sec x) = \sec x \tan x \longrightarrow \int \sec x \tan x\,dx = \sec x + C$

$\frac{d}{dx}(\csc x) = -\csc x \cot x \longrightarrow \int \csc x \cot x\,dx = -\csc x + C$

* THE ANTIDERIVATIVES FOR $\sec x$, $\tan x$, $\cot x$, $\csc x$ ARE HARD TO FIND *

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\left(\ln|\sec x + \tan x|\right)' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \sec x \left(\frac{\tan x + \sec x}{\sec x + \tan x}\right)$

INV. TRIG:

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C \quad \text{OR } -\arccos x + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

EX: $\int x^3 + 2\sin x - 4\,dx = \frac{1}{4}x^4 - 2\cos x - 4x + C$

EX: $\int \frac{x^2+1}{x}\,dx = \int x + \frac{1}{x}\,dx = \frac{1}{2}x^2 + \ln|x| + C$

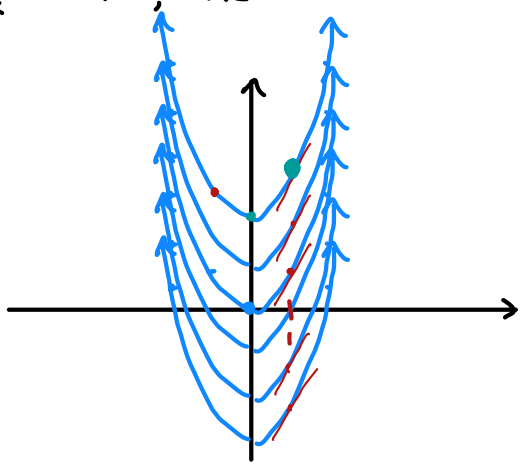
$\frac{x^2}{x} + \frac{1}{x}$

INITIAL VALUE PROBLEMS: IF WE HAVE A y-VALUE, WE CAN SOLVE FOR C AFTER INTEGRATING.

IF ALL WE KNOW IS $\frac{dy}{dx} = 2x$, THE BEST WE CAN DO IS

$$y = \int 2x\,dx$$

$$y = x^2 + C$$



EX: (I.V.P.)

$$\begin{cases} \frac{dy}{dx} = 2x \\ y(1) = 3 \end{cases}$$

POINT (1,3)

FIND y BY INTEGRATING $\frac{dy}{dx}$:

$$y = \int 2x\,dx = x^2 + C$$

USE $y(1) = 3$ TO GET C:

$$3 = 1^2 + C \longrightarrow C = 2$$

$$\boxed{y = x^2 + 2}$$

EX:

$$\begin{cases} \frac{d^2y}{dx^2} = -10 \\ y'(0) = 4 \\ y(0) = -3 \end{cases}$$

$$y' = \int -10\,dx$$

$$y' = -10x + C$$

$$4 = -10 \cdot 0 + C$$

$$C = 4$$

$$y' = -10x + 4$$

$$y = \int -10x + 4\,dx$$

$$= -5x^2 + 4x + C$$

$$-3 = 0 + 0 + C$$

$$C = -3$$

$$\boxed{y = -5x^2 + 4x - 3}$$