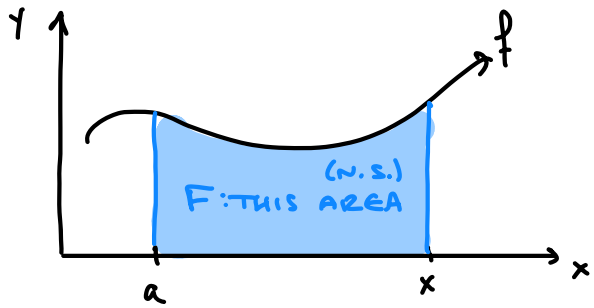
# MATH 122

THE FUNDAMENTAL THEOREM OF CALCULUS

GOALS

- ☐ BE ABLE TO STATE AND EXPLAIN BOTH PARTS OF THE FTC
- ☐ USE FTC TO EVALUATE DEFINITE INTEGRALS
- ☐ USE FTC TO SIMPLIFY EXPRESSIONS LIKE $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right)$
  - ☐ INCLUDE THE CHAIN RULE

DEFINE $F(x) = \int_a^x f(t)\,dt$

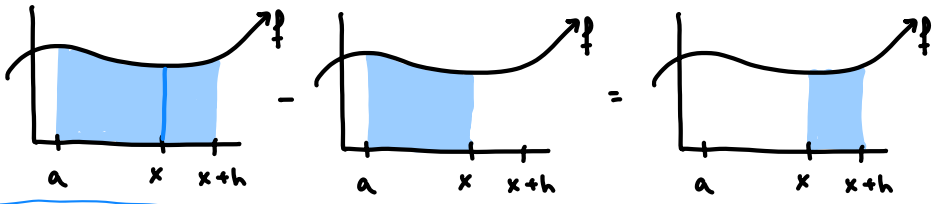



COMPUTE $F'$:

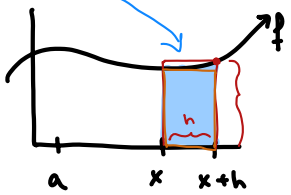
$$F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim \frac{1}{h}\left(\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt\right)$$

USE INTEGRAL PROPERTIES TO COMBINE THESE




$$= \lim_{h\to 0} \frac{1}{h}\int_x^{x+h} f(t)\,dt$$




$A = w \cdot h$

$A = h \cdot f(x+h)$

$A = h \cdot f(x)$

$$h\,f(x) \le \int_x^{x+h} f(t)\,dt \le h\,f(x+h)$$

$$(\div h)$$

$$f(x) \le \frac{1}{h}\int_x^{x+h} f(x)\,dt < f(x+h)$$

SINCE $\lim_{h\to 0} f(x) = f(x)$ AND $\lim_{h\to 0} f(x+h) = f(x)$ THE SQUEEZE THEOREM TELLS US $\lim_{h\to 0} \frac{1}{h}\int_x^{x+h} f(t)\,dt = f(x)$.

CONCLUSION: $$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$$

THIS MEANS THAT DIFFERENTIATION (SLOPE OF TANGENT) IS THE INVERSE OF INTEGRATION (AREA).

(LIKE HOW $\ln e^4 = 4$ ; $\arctan(\tan(\pi/3)) = \frac{\pi}{3}$ ; $\sqrt[3]{5^3} = 5$)

e.g. $\frac{d}{dx}\int_1^x e^{t^2}\,dt = e^{x^2}$.

PART 2: $$\int_a^b f(x)\,dx = \underbrace{(F(b)+C) - (F(a)+C)}_{\text{ACCOUNTS FOR POSSIBLE CONST.}} = F(b) - F(a) \text{ ; WHERE } F' = f$$

- F IS AN ANTIDERIVATIVE OF f

* CONSTANTS GET LOST WHEN WE TAKE A DERIV.

e.g. $\frac{d}{dx}(x^2) = 2x$, $\frac{d}{dx}(x^2+7) = 2x$ ...

- NOTATION: $F(x)\Big|_a^b = F(b) - F(a)$

e.g. $\int_0^2 x^2\,dx = F(x)\Big|_0^2$ where $F'(x) = x^2$, so $F(x) = \frac{1}{3}x^3$

$= \frac{1}{3}x^3\Big|_0^2$

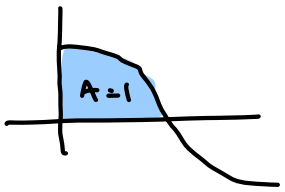
$= \frac{1}{3}2^3 - \frac{1}{3}\cdot 0^3$

$= \boxed{\frac{8}{3}}$

Ex: $\int_0^{\pi/2} \cos(x)\,dx = \sin(x)\Big|_0^{\pi/2}$ B/c $\frac{d}{dx}(\sin x) = \cos x$

$= \sin\frac{\pi}{2} - \sin 0$

$= \boxed{1}$




## TRICKIER PART 1 QUESTIONS

Ex: $\frac{d}{dx}\int_x^1 \tan(t^2)\,dt$ THIS DOESN'T FIT OUR FORM $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

USE PROPERTY: $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ B/C $\Delta x < 0$ IF GOING BACKWARDS.

$\frac{d}{dx}\int_x^1 \tan(t^2)\,dt = -\frac{d}{dx}\int_1^x \tan(t^2)\,dt$

$= \boxed{-\tan(x^2)}$

Ex: $\frac{d}{dx}\int_2^{x^3}\sqrt{t^4+1}\,dt$

$= 3x^2\sqrt{x^{12}+1}$

* PLUG IN $x^3$ FOR AND MULT. BY DERIV. OF $x^3$. *

Ex: $\frac{d}{dx}\int_0^{\tan x} e^t\,dt = \boxed{\sec^2 x\, e^{\tan x}}$

$F(x) = \int_2^x \sqrt{t^4+1}\,dt$

$\boxed{F'(x) = \sqrt{x^4+1}}$

$F(x^3) = \int_2^{x^3}\sqrt{t^4+1}\,dt$

$\frac{d}{dx}\left(F(x^3)\right) = F'(x^3)\cdot 3x^2$

CHAIN RULE

$= \sqrt{(x^3)^4+1}\cdot 3x^2$