MATH 121

DIFFERENTIATION TECHNIQUES

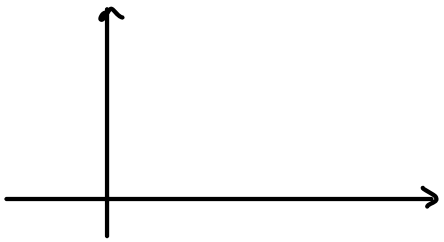
GOALS

- USE THE LIMIT DEFINITION OF THE DERIVATIVE TO FIND "SHORTCUTS" FOR COMPUTING THE DERIVATIVES OF CONSTANTS, POWER FUNCTIONS, AND TRIG FUNCTIONS
- USE THE LIMIT DEFINITION OF THE DERIVATIVE TO COME UP WITH RULES FOR COMBINING DERIVATIVES
  - SUMMATION RULE, CONSTANT MULTIPLE RULE, PRODUCT RULE, AND QUOTIENT RULE

NOTATION $\frac{d}{dx}$ MEANS "THE DERIVATIVE OF ..." (INCOMPLETE PHRASE)

$$\text{e.g. } \frac{d}{dx}\left(2\sqrt{x}\right) = \frac{1}{\sqrt{x}}$$

CONSTANT FUNCTIONS $f(x) = k$



$$\text{e.g. } g(x) = 7$$

$$h(x) = 13\pi^3$$

$$i(x) = e^{23}$$

POWER FUNCTIONS $f(x) = x^n$ (n IS A POSITIVE INTEGER)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

PASCAL'S TRIANGLE

e.g $g(x) = x^6$

$h(x) = \sqrt[3]{x} = x^{1/3}$

$i(x) = \frac{1}{x^4} = x^{-4}$

COMBINING DERIVATIVES:

CONSTANT MULTIPLE

$$\frac{d}{dx}\left(k f(x)\right) =$$

e.g. $y = 3x^7$

SUM / DIFFERENCE

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) =$$

e.g. $h(x) = 3x^7 + 2\sqrt{x}$

PRODUCT RULE

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x)g(x) + f(x)g'(x)$$

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$$

e.g. $y = \dfrac{4x+1}{x^2+3}$

SUMMARY

CONSTANT RULE

$$\frac{d}{dx}(k) =$$

POWER RULE

$$\frac{d}{dx}(x^n) =$$

CONST. MULTIPLE RULE

$$\frac{d}{dx}(kf(x)) =$$

SUM/DIFFERENCE RULE

$$\frac{d}{dx}(f(x) \pm g(x)) =$$

PRODUCT RULE

$$\frac{d}{dx}(f(x)g(x)) =$$

QUOTIENT RULE.

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$$

TRY: COMPUTE THE DERIVATIVE.

① $f(x) = 2x^5 - x^3 + 4x - 1$

③ $g(x) = \dfrac{4}{\sqrt{x}}$

③ $h(x) = \dfrac{2x^2 - x + 7}{x}$

TRIG

$f(x) = \sin x$ $\qquad f'(x) = \lim_{h \to 0} \dfrac{\sin(x+h) - \sin x}{h}$

MAIN TWO:

$\dfrac{d}{dx}(\sin x) =$

$\dfrac{d}{dx}(\cos x) =$

Ex: $y = x^2 \sin x$

OTHER FOUR

$$\frac{d}{dx}(\tan x) =$$

$$\frac{d}{dx}(\sec x) =$$

TRIG DERIVATIVES

$\frac{d}{dx}(\sin x) =$ $\qquad$ $\frac{d}{dx}(\cos x) =$

$\frac{d}{dx}(\tan x) =$ $\qquad$ $\frac{d}{dx}(\sec x) =$

$\frac{d}{dx}(\cot x) =$ $\qquad$ $\frac{d}{dx}(\csc x) =$

TRY: FOR 1-3, COMPUTE THE DERIVATIVE

① $f(x) = x^3 \sin x$ ② $g(x) = \frac{x+1}{\tan x}$ ③ $h(x) = \sin x \cos x$

④ FIND AN EQUATION FOR THE TANGENT LINE TO $y = \tan x$ AT $x = \frac{3\pi}{4}$

⑤ FIND ALL X-VALUES WHERE $y = 2\sin x - x$ HAS A HORIZONTAL TANGENT LINE.

① $f(x) = \underset{A}{x^3} \underset{B}{\sin x}$

② $g(x) = \frac{x+1}{\tan x}$

③ $h(x) = \sin x \cos x$

④ FIND AN EQUATION FOR THE TANGENT LINE TO $y = \tan x$ AT $x = \frac{3\pi}{4}$

⑤ FIND ALL X-VALUES WHERE $y = 2\sin x - x$ HAS A HORIZONTAL TANGENT LINE.