MATH 122

DEFINITE INTEGRALS

GOALS

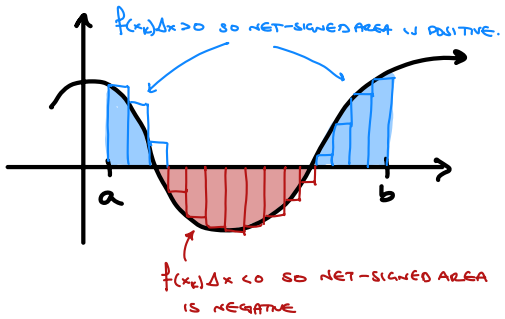
- ☐ INTERPRET DEFINITE INTEGRALS AS NET SIGNED AREAS.
- ☐ USE FAMILIAR FORMULAS FROM GEOMETRY TO EVALUATE DEFINITE INTEGRALS
- ☐ EVALUATE A DEFINITE INTEGRAL FROM A GRAPH BY APPLYING PROPERTIES OF INTEGRALS

NET-SIGNED AREA:

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

$f(x_k^*)$: HEIGHTS — CAN BE NEGATIVE!

Δx: WIDTHS — ALWAYS POSITIVE: MOVING LEFT TO RIGHT MEANS x IS INCREASING SO $\Delta x > 0$. ALSO, $b > a$ IN $[a,b]$ AND $\frac{b-a}{n} > 0$

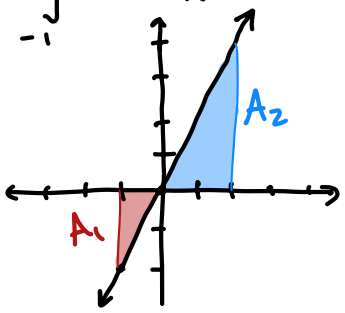


NEW NOTATION: DEFINITE INTEGRAL.

$$\int_a^b f(x)\,dx$$: NET-SIGNED AREA FOR f OVER $[a,b]$

- $\int_a^b$ TAKES PLACE OF $\lim \sum$ AND $[a,b]$
- dx: DIFFERENTIAL TERM
  - TAKES PLACE OF Δx AND STANDS FOR "INFINITELY SMALL CHANGE IN x"
  - TELLS US WHAT THE VARIABLE IS (AS IN $\frac{dy}{dx}$)

Ex: $\int_{-1}^{2} 2x\,dx$: NET-SIGNED AREA FOR $f(x) = 2x$ OVER $[-1,2]$



$A_1 = \frac{1}{2} \cdot 1 \cdot 2 = 1$

$A_2 = \frac{1}{2} \cdot 2 \cdot 4 = 4$

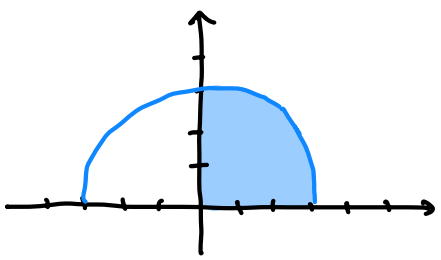
$$\int_{-1}^{2} 2x\,dx = \underset{A_2}{4} - \underset{A_1}{1} = 3$$

WHY WE MIGHT WANT N.S.A.: AVERAGE OF A CONTINUOUS FUNCTION

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

e.g. $\frac{1}{2-(-1)}\int_{-1}^{2} 2x\,dx = \frac{1}{3}\cdot 3 = \boxed{1}$

Ex: $\int_0^3 \sqrt{9-x^2}\,dx$: N.S.A. FOR $f(x) = \sqrt{9-x^2}$ ON $[0,3]$.



$y = \sqrt{9-x^2}$ TOP HALF OF CIRCLE.

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$ ← $x^2+y^2=r^2$

CIRCLE W/ RADIUS 3

$$\int_0^3 \sqrt{9-x^2}\,dx = \frac{1}{4}\pi(3)^2 = \boxed{\frac{9\pi}{4}}$$

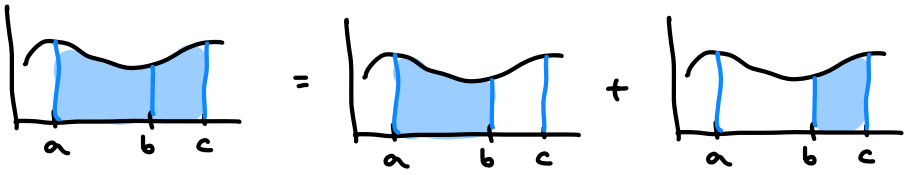
PROPERTIES:

① $\int_a^b f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$ | $\Sigma f+g = \Sigma f + \Sigma g$

② $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$ | $\Sigma c f(x) = c\,\Sigma f(x)$

③ $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ | IF GOING RIGHT TO LEFT, $\Delta x < 0$.

④ $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

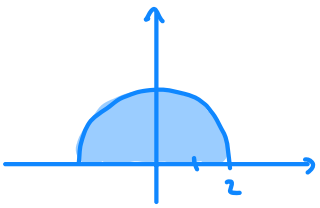


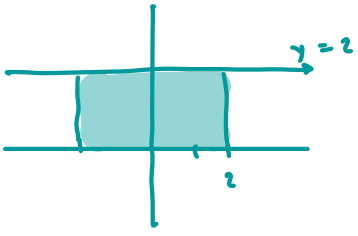
Ex: $\int_{-2}^{2}(3\sqrt{4-x^2} + 2)\,dx = \int_{-2}^{2} 3\sqrt{4-x^2}\,dx + \int_{-2}^{2} 2\,dx$

$= 3\int_{-2}^{2}\sqrt{4-x^2}\,dx + \int_{-2}^{2} 2\,dx$

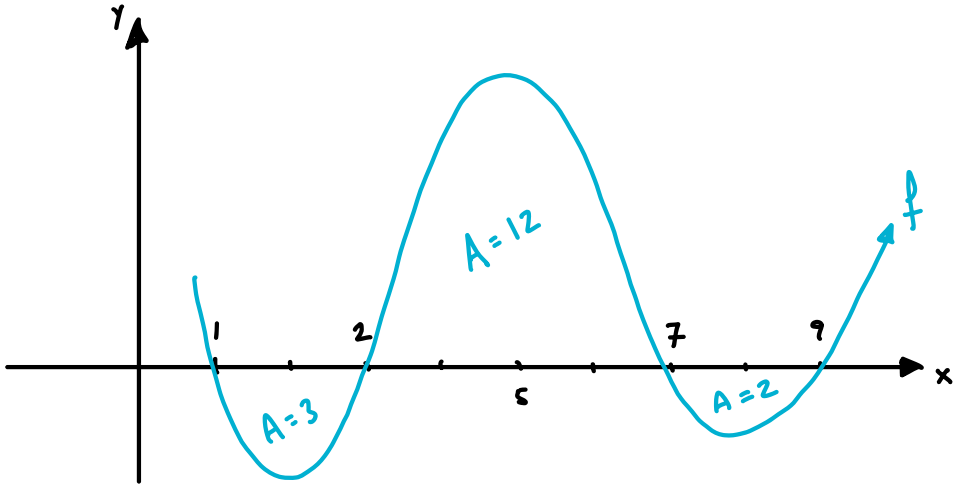
$= 3\left(\frac{1}{2}\pi(2)^2\right) + 2\cdot 4$

$= \boxed{6\pi + 8}$





Ex: USE THE GRAPH OF f TO EVALUATE THE FOLLOWING DEFINITE INTEGRALS.



$$\int_1^9 f(x)\,dx = 7$$

$$\int_1^7 f(x)\,dx = 9$$

$$\int_9^2 f(x)\,dx = -10$$

$$\int_1^9 |f(x)|\,dx = 17$$