MATH 121

CONTINUITY

GOALS

- ☐ DEFINE CONTINUITY AT A POINT (3 CONDITIONS)
- ☐ DEFINE LEFT- AND RIGHT-CONTINUITY
- ☐ APPLY DEFINITIONS OF CONTINUITY VISUALLY
- ☐ EXTEND OUR DEFINITIONS TO CONTINUITY ON INTERVALS
- ☐ GIVEN A PIECEWISE FUNCTIONS WITH SOME UNSPECIFIED PARAMETERS, CHOOSE VALUES FOR THOSE PARAMETERS THAT MAKE THE FUNCTION CONTINUOUS

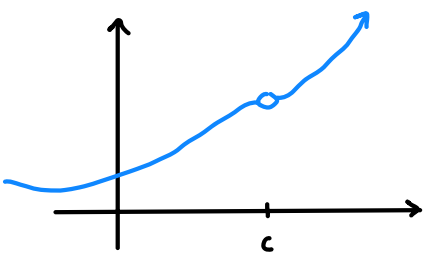
DEFINITION

A FUNCTION f IS CONTINUOUS AT $x = a$ IF...

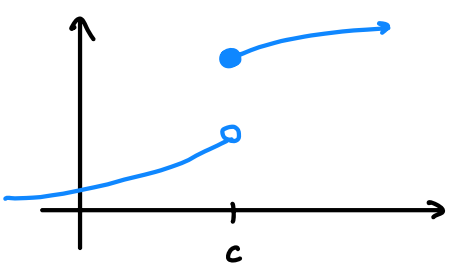
1.
2.
3.

EXAMPLES OF DISCONTINUITIES

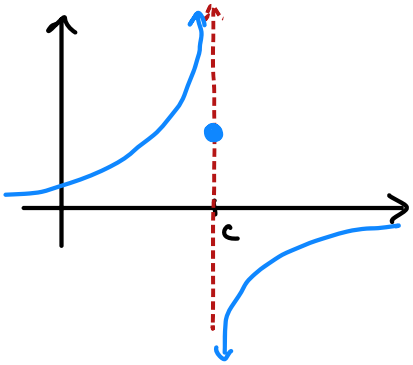
1)



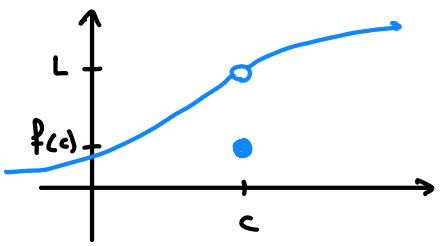
2)



3)



4)



CHECKING FOR CONTINUITY

NON-PIECEWISE

e.g. $f(x) = \dfrac{2x-1}{x^2-16}$

$$g(x) = \frac{1}{e^x + 1}$$

PIECEWISE: BE CAREFUL WHERE DEFINITION CHANGES.

$$f(x) = \begin{cases} x^2 - 1 \; ; & x < -1 \\ \frac{1}{2}x \; ; & -1 \leq x \leq 4 \\ \sqrt{x} \; ; & x > 4 \end{cases}$$

CONTINUITY ON INTERVALS

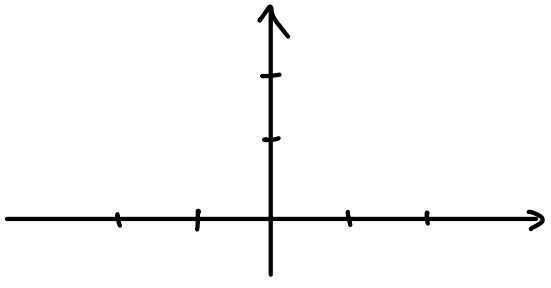
OPEN: (a, b) OR $a < x < b$ OR 

CLOSED: $[a, b]$ OR $a \leq x \leq b$ OR 

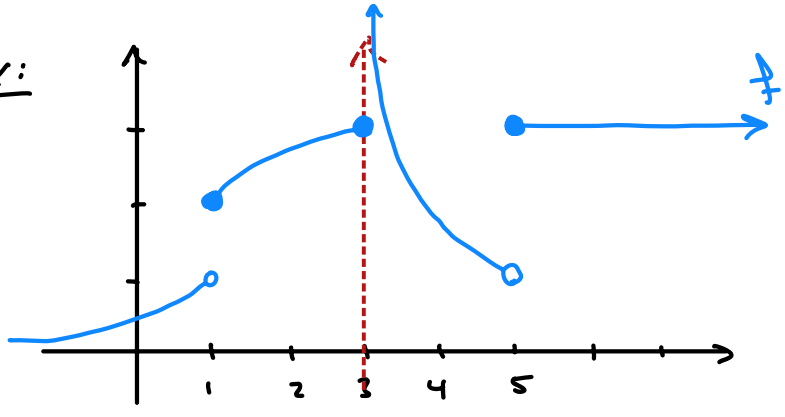
* **CONTINUOUS ON (a, b)**:

* CONT. ON $[a, b]$:

e.g. $f(x) = \sqrt{4 - x^2}$



EX:



INTERMEDIATE VALUE THEOREM

APPLYING IVT

e.g. $f(x) = x^5 + x^4 + x - 1$

SHOW f HAS A ROOT IN $[0,1]$

BONUS EXERCISE

FIND VALUES FOR a AND b THAT MAKE f CONTINUOUS EVERYWHERE

$$f(x) = \begin{cases} \frac{1}{x} \; ; & x \leq -1 \\ ax + b \; ; & -1 < x \leq 2 \\ x^2 + 1 \; ; & x > 2 \end{cases}$$