MATH 121

CONTINUITY

GOALS

- ☐ DEFINE CONTINUITY AT A POINT (3 CONDITIONS)
- ☐ DEFINE LEFT- AND RIGHT-CONTINUITY
- ☐ APPLY DEFINITIONS OF CONTINUITY VISUALLY
- ☐ EXTEND OUR DEFINITIONS TO CONTINUITY ON INTERVALS
- ☐ GIVEN A PIECEWISE FUNCTIONS WITH SOME UNSPECIFIED PARAMETERS, CHOOSE VALUES FOR THOSE PARAMETERS THAT MAKE THE FUNCTION CONTINUOUS

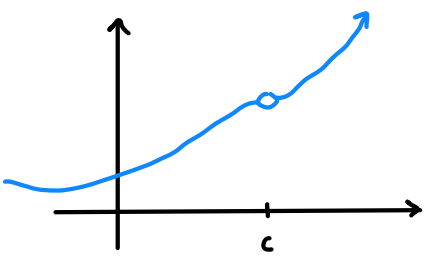
DEFINITION

A FUNCTION f IS CONTINUOUS AT $x = a$ IF...

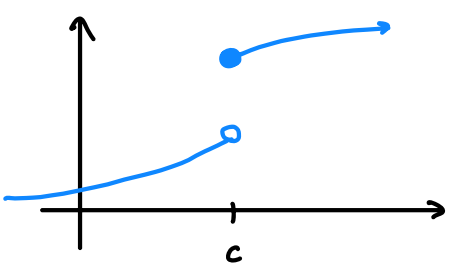
1. $f(a)$ IS DEFINED
2. $\lim_{x \to a} f(x)$ EXISTS
3. $\lim_{x \to a} f(x) = f(a)$

EXAMPLES OF DISCONTINUITIES

1)



2)



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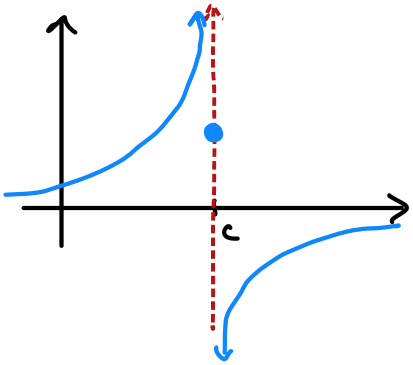
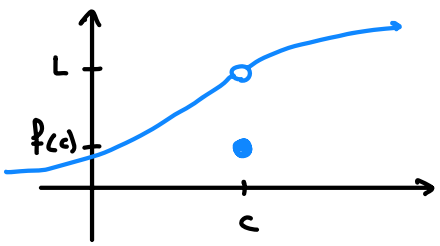


4)



CHECKING FOR CONTINUITY

NON-PIECEWISE JUST CHECK DOMAIN.

e.g. $f(x) = \dfrac{2x-1}{x^2-16}$

$x^2 - 16 = 0 \longrightarrow x = \pm 4$

f IS DISCONT. AT $x = \pm 4$

$g(x) = \dfrac{1}{e^x + 1}$

$e^x + 1 = 0$

$e^x = -1$

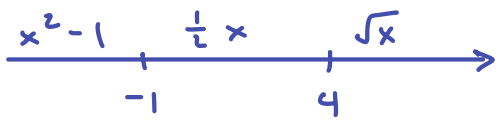
NO SOL'N

$e^{-2} = \dfrac{1}{e^2}$ ✓

g HAS NO DISCONTINUITY

PIECEWISE: BE CAREFUL WHERE DEFINITION CHANGES.

$$f(x) = \begin{cases} x^2 - 1 & ; \quad x < -1 \\ \frac{1}{2}x & ; \quad -1 \le x \le 4 \\ \sqrt{x} & ; \quad x > 4 \end{cases}$$



$x = -1$

① $f(-1) = \frac{1}{2}(-1) = -\frac{1}{2}$ ✓

② $\lim\limits_{x \to -1^-} f(x) = (-1)^2 - 1 = 0$

$\lim\limits_{x \to -1^+} f(x) = \frac{1}{2}(-1) = -\frac{1}{2}$

$\lim\limits_{x \to -1} f$ DNE ✗

f IS DISCONT. AT $x = -1$

$x = 4$

① $f(4) = \frac{1}{2}(4) = 2$ ✓

② $\lim\limits_{x \to 4^-} f(x) = \frac{1}{2}(4) = 2$

$\lim\limits_{x \to 4^+} f(x) = \sqrt{4} = 2$ ✓

$\lim\limits_{x \to 4} f = 2$ ✓

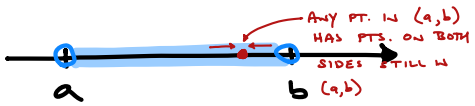
f IS CONT. AT $x = 2$

CONTINUITY ON INTERVALS

OPEN: (a,b) OR $a < x < b$ OR 

CLOSED: $[a,b]$ OR $a \le x \le b$ OR 

* **CONTINUOUS ON (a,b)**: f IS CONTINUOUS AT EACH POINT IN (a,b)



* CONT. ON $[a,b]$: f IS CONT. ON (a,b) AND

$$\lim_{x \to a^+} f = f(a) \quad \& \quad \lim_{x \to b^-} f = f(b).$$

e.g. $f(x) = \sqrt{4 - x^2}$

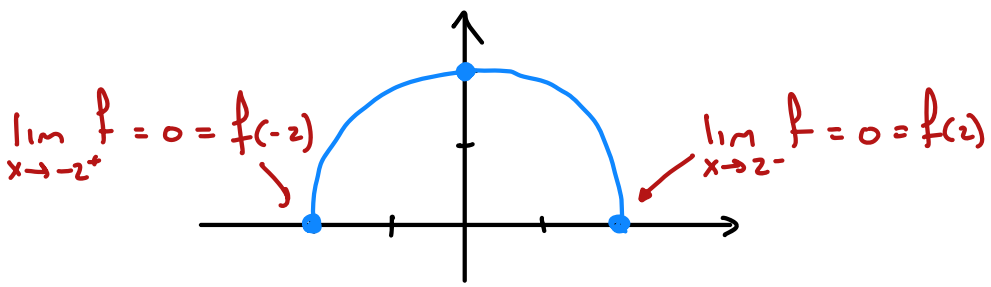
TOP HALF

$y = \sqrt{4 - x^2}$

$y^2 = 4 - x^2$

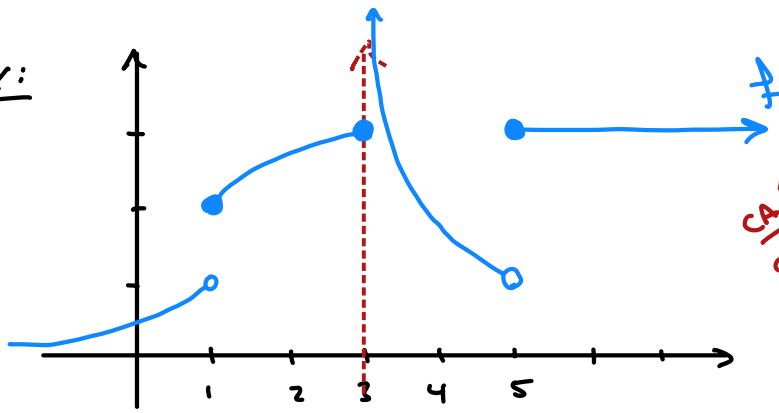
$x^2 + y^2 = 4$

CIRCLE:
RADIUS: 2
CENTER: (0,0)



f IS CONTINUOUS ON $[-2,2]$

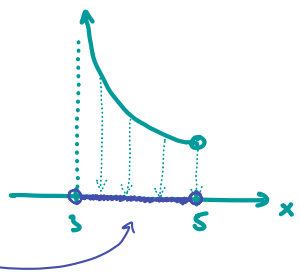
EX:



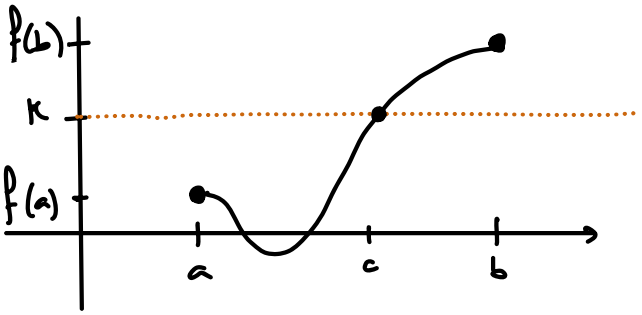
f IS CONT. ON ...

CAN'T BE COMBINED

- $[1,3]$
- $[5,\infty)$
- $(-\infty,1)$
- $(3,5)$



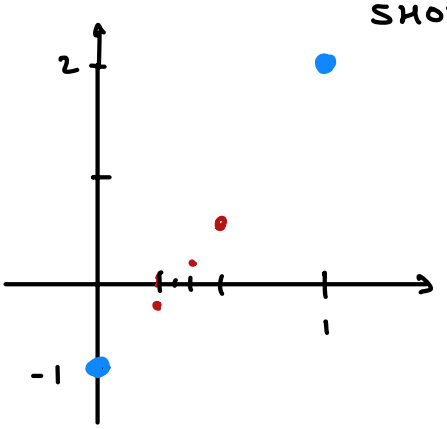
INTERMEDIATE VALUE THEOREM IF f IS CONTINUOUS ON $[a,b]$ THEN FOR ANY y-VALUE K BETWEEN $f(a)$ AND $f(b)$ THERE MUST BE SOME x-VALUE c SO THAT $f(c) = K$.



APPLYING IVT FINDING ROOTS / ZEROS.

e.g. $f(x) = x^5 + x^4 + x - 1$

SHOW f HAS A ROOT IN $[0,1]$



- f IS CONTINUOS
- $f(0) = -1$
- $f(1) = 2$
- SINCE $f(0) < 0 < f(1)$, FOR SOME c IN $(0,1)$ $f(c) = 0$.

BONUS EXERCISE FIND VALUES FOR a AND b THAT MAKE f CONTINUOUS EVERYWHERE

$$f(x) = \begin{cases} \frac{1}{x} \; ; \; x \le -1 & \frac{1}{x} \text{ IS CONT. ON } (-\infty, -1) \\ ax+b \; ; \; -1 < x \le 2 & ax+b \text{ IS CONT. ON } (-1, 2) \\ x^2+1 \; ; \; x > 2 & x^2+1 \text{ IS CONT ON } (2, \infty) \end{cases}$$

$x = -1$ $f(-1) = \frac{1}{-1} = -1$ ✓

$$\left.\begin{aligned} \lim_{x \to -1^-} f(x) &= \lim_{x \to -1^-} \frac{1}{x} = -1 \\ \lim_{x \to -1^+} f(x) &= \lim_{x \to -1^+} ax+b = -a+b \end{aligned}\right\} \; -a+b = -1$$

x = 2

$f(2) = 2a + b$ ✓

$\lim_{x \to 2^-} f(x) = 2a + b$

$\lim_{x \to 2^+} f(x) = 2^2 + 1 = 5$

$\Big\}$ $2a + b = 5$

$$-a + b = -1$$
$$2a + b = 5$$
$$\overline{\qquad\qquad}$$
$$3a = 6$$

$\boxed{a = 2} \longrightarrow -2 + b = -1$

$\boxed{b = 1}$