MATH 121

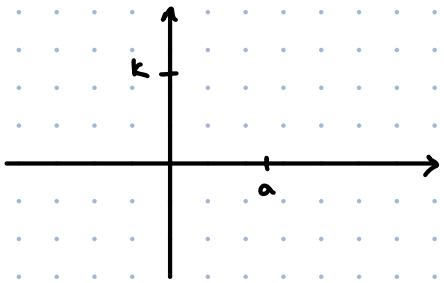
COMPUTING LIMITS

GOALS

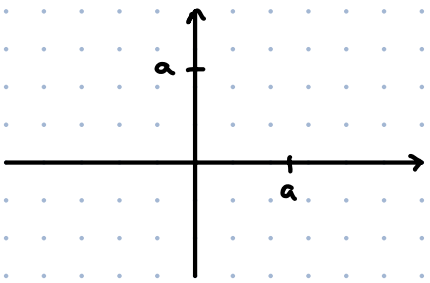
- ☐ GIVEN A FUNCTION, COMPUTE THE ONE-SIDED AND TWO-SIDED LIMITS AS x APPROACHES A FINITE VALUE
- ☐ KNOW WHAT TO DO FOR EACH OF THREE CASES FOR NON PIECEWISE FUNCTIONS
  1. PLUG IN AND DONE
  2. GET $\frac{0}{0}$
  3. GET $\frac{k}{0}$ FOR $k \neq 0$
- ☐ APPLY THE THREE CASES AND OUR ISTUITIVE APPROACH TO PIECEWISE FUNCTIONS

SOME IMPORTANT PROPERTIES OF LIMITS

1. $\lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) =$
2. $\lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) =$
3. $\lim_{x \to a} f(x) \Big/ \lim_{x \to a} g(x) =$
4. $\left(\lim_{x \to a} f(x)\right)^n =$
5. $\lim_{x \to a} k =$



6. $\lim_{x \to a} x =$



EX SUPPOSE $\lim_{x \to a} f(x) = 4$ AND $\lim_{x \to a} g(x) = 7$

1. $\lim_{x \to a} \left(f(x) + g(x) \right) =$

2. $\lim_{x \to a} 2 f(x) =$

3. $\lim_{x \to a} \left(3 f(x) - 2 g(x) \right) =$

THREE CASES $\lim_{x \to a} f(a)$ WHERE f IS NOT PIECEWISE

1. $f(a)$ IS DEFINED (I.E. WE CAN PLUG IN $x = a$ DIRECTLY)

2. PLUG IN $x = a$ AND GET $\frac{0}{0}$

3. PLUG IN $x = a$ AND GET $\frac{k}{0}$ $(k \neq 0)$

CASE 1

IF $f(a)$ IS DEFINED, THEN

EXAMPLES

1. $\lim_{x \to 3} (x^2 + 1) =$

2. $\lim_{x \to -1} \ln(x + 2) =$

3. $\lim_{x \to \pi} \sin(x/2) =$

4. $\lim_{x \to 0} e^x =$

CASE 2

IF WE PLUG $x = a$ AND GET $\frac{0}{0}$, THEN WE CAN SIMPLIFY BY

EXAMPLES

1. $\lim_{x \to 4} \dfrac{x - 4}{x^2 - 16}$

2. $\lim_{x \to -1} \dfrac{x^2 + 3x + 2}{x^2 - 2x - 3}$

3. $\lim_{x \to 3} \dfrac{x^2 - x - 6}{x^2 - 9}$

4. $\lim_{x \to 9} \dfrac{x - 9}{\sqrt{x} - 3}$

TWO OPTIONS FOR HOW TO HANDLE THIS

i

ii.

i.

ii.

5. $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

CASE 3

IF WE PLUG $x = a$ AND GET $\frac{k}{0}$ $(k \neq 0)$, THEN THE GRAPH OF f HAS A VERTICAL ASYMPTOTE AT $x = a$.

* THIS MEANS $\lim_{x \to a^-} f(x) = \pm$ AND $\lim_{x \to a^+} f(x) = \pm\infty$.

WE CAN USE SIGN CHARTS TO FIGURE OUT THE SIGNS.

EXAMPLES

1. $\lim_{x \to 2^-} \frac{3x - 5}{x - 2}$

2. $\lim_{x \to -1^+} \frac{2x + 1}{x^2 - 1}$

SOME MORE EXAMPLES

1. $\lim_{x \to 3^+} \frac{x-1}{|x-3|}$

2. $\lim_{x \to 1} \frac{x^2+3x+4}{x^2-x-2}$

3. $\lim_{x \to -2} \frac{x^2+5x+6}{x^2+6x+8}$

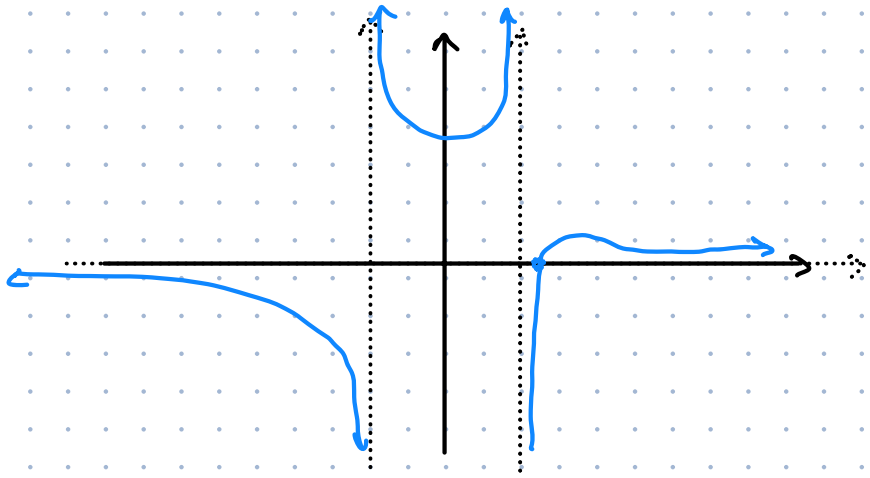
4. $\lim_{x \to 25} \frac{\sqrt{x}-5}{x-25}$

5. $\lim_{x \to 1^+} \frac{10x - 11}{x^2 - 1} =$

$\lim_{x \to 1^-} \frac{10x - 11}{x^2 - 1} =$

$\lim_{x \to -1^-} \frac{10x - 11}{x^2 - 1} =$

$\lim_{x \to -1^+} \frac{10x - 11}{x^2 - 1} =$



WORKING W/ PIECEWISE FUNCTIONS

$$f(x) = \begin{cases} g(x) ; & x \leq a \\ h(x) ; & x > a. \end{cases}$$

e.g. $$f(x) = \begin{cases} \frac{1}{x+2} ; & x < -2 \\ x + 1 ; & -2 \leq x < 1 \\ \sqrt{x+3} ; & x \geq 1 \end{cases}$$