MATH 122

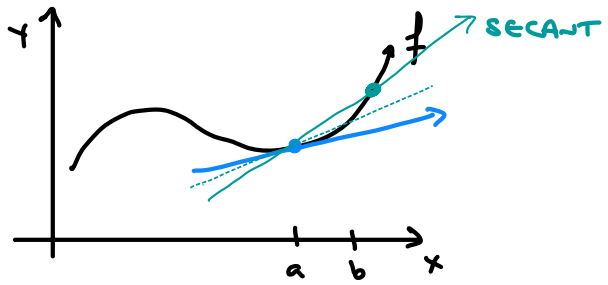
AREA AS A LIMIT

GOALS

- ☐ APPROXIMATE THE AREA UNDER A CURVE USING A GIVEN NUMBER OF RECTANGLES.
  - ☐ KNOW HOW TO DO SO WITH LEFT ENDPOINTS, RIGHT ENDPOINTS AND MID POINTS.
- ☐ WORK WITH SUMMATION NOTATION
  - ☐ EXPAND A GIVEN SUM
  - ☐ WRITE A SUM IN SIGMA NOTATION
  - ☐ APPLY SUMMATION FORMULAS TO COMPUTE LARGE SUMS
- ☐ USE RIEMANN SUMS TO COMPUTE EXACT AREA UNDER A CURVE
  - ☐ KNOW THE FORMULA AND INDIVIDUAL TERMS $(\Delta x, x_k^*)$

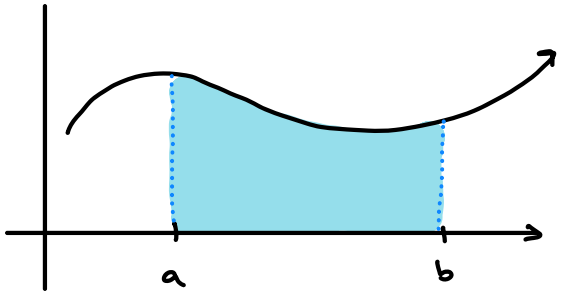
CALC 1 RECAP:

Q: HOW DO WE COMPUTE THE SLOPE OF THE TANGENT LINE TO f AT $x=a$?



CALC 2:

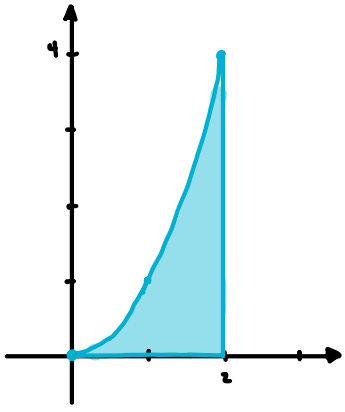
Q: GIVEN A CONTINUOUS f, HOW DO WE COMPUTE THE AREA BETWEEN THE GRAPH OF f AND THE x-AXIS OVER $[a,b]$?



- APPROXIMATION: FILL AREA w/ RECTANGLES (EASIEST AREA)

  EXACT: LET NUMBER OF RECT. GO TO INFINITY (LIMITS)

EX: $f(x) = x^2$; $[0,2]$; USE 4 RECTANGLES



RECTANGLES:

- EQUAL WIDTHS:

- USE TOP-RIGHT CORNER TO DETERMINE HEIGHTS.

$A \approx$

SUMMATION / SIGMA NOTATION

$$\sum_{k=1}^{n} f(k)$$

e.g. $\sum_{k=1}^{5} \underbrace{(2k-1)}_{f(k)} =$

EX: WRITE $1+4+9+16+25+36+49$ IN Σ-NOTATION

EX: WRITE $6-9+12-15+18-21+24-27$ IN Σ-NOTATION

EX: WRITE $1+3+5+7+\cdots+21$ IN Σ-NOTATION

PROPERTIES:

① $\sum_{k=1}^{n} (f(k) + g(k)) =$

② $\sum_{k=1}^{n} a f(k) =$

FORMULAS:

③ $\sum_{k=1}^{n} 1 =$

④ $\sum_{k=1}^{n} k =$

e.g. $\sum_{k=1}^{100} k =$

⑤ $\sum_{k=1}^{n} k^2 =$

⑥ $\sum_{k=1}^{n} k^3 =$

Ex: $\sum_{k=1}^{24} (2k-1) =$

Ex: $\sum_{k=1}^{12} (k^2 - 2k) =$

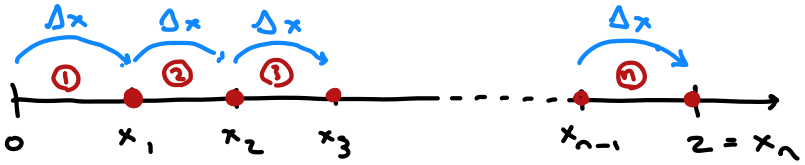
Ex: $\sum_{k=4}^{20} (3k+1) =$

EX: $f(x) = x^2$; $[0,2]$; USE n RECTANGLES w/ RIGHT ENDPOINTS.

$$\Delta x = \frac{\text{(LENGTH OF INTERVAL)}}{\text{(NUMBER OF RECT.)}}$$

$$\boxed{\Delta x =}$$

X-VALUES: x_k^* OR x_k



COMPARE w/ WHAT WE DID WHEN $n=4$

$\Delta x = \frac{2}{4} = \frac{1}{2}$

$x_k = \frac{2k}{4} = \frac{k}{2}$

$x_1 = \frac{1}{2}$, $x_2 = 1$

$x_3 = \frac{3}{2}$; $x_4 = 2$

$x_1 =$

$x_2 =$

$\vdots$

$x_k =$

$\vdots$

$x_n =$

HEIGHTS: PLUG x_k INTO GIVEN FUNCTION $f(x)$

$$f(x_k) =$$

ADD UP ALL n AREAS:

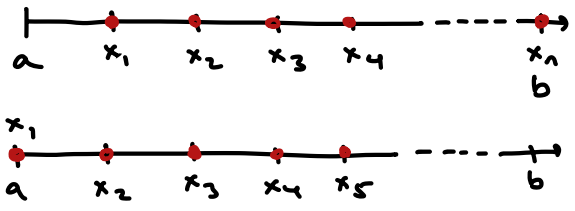
$$A \approx$$

TAKE LIMIT AS $n \to \infty$ TO GET EXACT AREA:

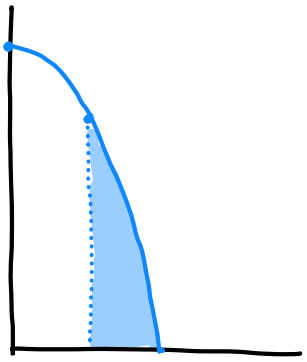
$$A =$$

IN GENERAL: THE NET-SIGNED AREA FOR f OVER $[a,b]$ IS GIVEN BY THE RIEMANN SUM

$$x_K = \begin{cases} \text{RIGHT:} \\ \text{LEFT:} \\ \text{MID:} \end{cases}$$



Ex: $f(x) = 4 - x^2$; $[1, 2]$; RIGHT ENDPOINTS



$\Delta x =$

$x_k =$

$f(x_k) =$

$A =$

ex: $f(x) = x^3$; $[0, 1]$; LEFT ENDPOINTS

$\Delta x =$ $\qquad$ $x_k^* =$

$f(x_k^*) =$

$f(x_k^*) \cdot \Delta x =$

$=$

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*) \Delta x$$