MATH 121

ANALYSIS OF FUNCTIONS

GOALS

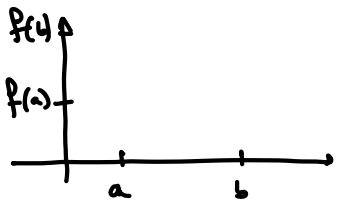
- ☐ BE ABLE TO DESCRIBE THE RELATIONSHIPS BETWEEN f, f', AND f''
- ☐ DETERMINE THE INTERVALS ON WHICH f IS INCREASING, DECREASING, CONCAVE UP, AND CONCAVE DOWN.
- ☐ FIND RELATIVE EXTREMA FOR A FUNCTION f
- ☐ FIND INFLECTION POINTS FOR A FUNCTION f
- ☐ FIND ABSOLUTE EXTREMA FOR A FUNCTION f

CENTRAL QUESTION: WHAT DOES f' TELL US ABOUT f? AND f''?

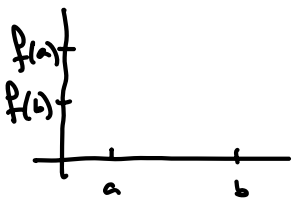
RECALL:

$$f'(a) = \lim_{b \to a} \frac{f(b) - f(a)}{b - a} \qquad (b > a)$$

INCREASING:



DECREASING:



How about f''?

$f'' > 0 \longrightarrow f'$ is $\longrightarrow f$.

$f'' < 0 \longrightarrow f'$ is $\longrightarrow f$

Ex: $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 1$

- Find intervals where f is increasing and decreasing.
  - Find f':
  - Identify points where f' can change sign
    1. $f' = 0$:
    2. f' undefined:
  - Make sign chart: choose a "test point" in each interval.
  - Conclusions:

· FIND INTERVALS WHERE f IS CONCAVE UP AND CONCAVE DOWN.

- FIND f'':
- FIND POINTS WHERE f'' CAN CHANGE SIGN:
    1. $f'' = 0$:
    2. f'' UNDEF:
- SIGN CHART:

- CONCLUSIONS: f IS CONCAVE UP ON

  CONCAVE DOWN ON

EX: $f(x) = 2x + 3x^{2/3}$

· INC/DEC:

- FIND $f'(x)$:
- f' CAN CHANGE SIGN:
    1. $f' = 0$:

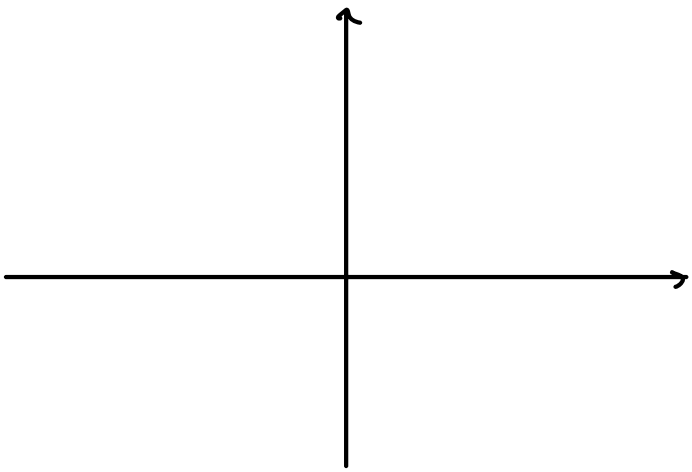
    2. f' UNDEF:

- SIGN CHART:

- **CONCAVITY**:
  - FIND f'' :
  - f'' CHANGES SIGN:
    1. $f'' = 0$:
    2. f'' UNDEF :
  - **SIGN CHART**:

SKETCH

$f(x) = 2 + 3x^{2/3}$





f IS...

INC :

DEC :

CONC. UP :

CONC DOWN :

WE ALSO CAN SEE THAT f HAS A

RELATIVE MAX AT

RELATIVE MIN AT

RECAP

$f' > 0 \longrightarrow f$

$f' < 0 \longrightarrow f$

$f'' > 0 \longrightarrow f' \longrightarrow f$

$f'' < 0 \longrightarrow f \longrightarrow f$

PROCESS:

1.

2.

3.

4.

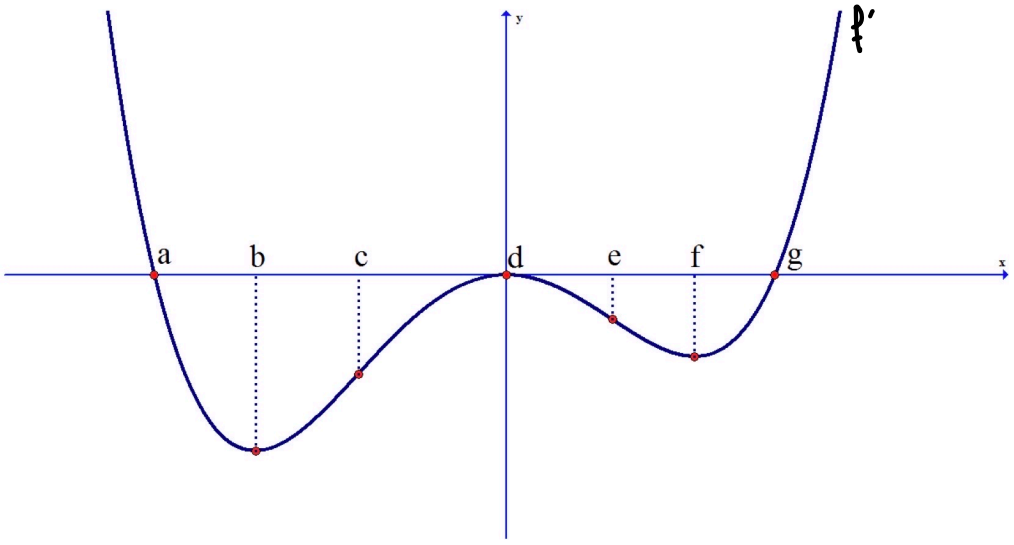
EXTREMA:

- f HAS REL. MAX IF IT GOES FROM
- f HAS REL. MIN IF IT GOES FROM
- f HAS AN INFLECTION POINT IF IT

Ex: $f(x) = xe^x$

TRY:

$$f(x) = \frac{2x}{x^2+1}$$



f IS INC:

f IS DEC:

REL. MAX:

REL MIN:

CONC. UP:

CONC DOWN:

TRY:

$$f(x) = x^4 - 2x^2$$

FIND WHERE f IS INCREASING/DECREASING, CONCAVE UP/DOWN, HAS RELATIVE EXTREMA, AND HAS INFLECTION POINTS.

ABSOLUTE EXTREMA:

EXTREME VALUE THEOREM: IF f IS CONTINUOUS ON $[a,b]$, THEN IT MUST HAVE AN ABSOLUTE MAX AND AN ABSOLUTE MIN ON $[a,b]$ AT EITHER A CRITICAL POINT OR AN END POINT.

- WHY CONTINUOUS?

- WHY CLOSED INTERVAL $[a,b]$?

- OTHERWISE ...

Ex: $f(x) = x^4 - 2x^2 \; ; \; [0, 2]$

- GET $f'(x)$:
- FIND CRITICAL PTS IN $[0,2]$

- EVALUATE f AT CRIT. PTS. AND END PTS:

Ex: $g(x) = x^2 e^x \; ; \; [-3, 1]$

ABSOLUTE EXTREMA RECAP:

- FOR CONTINUOUS f ON $[a,b]$:

- FOR CONTINUOUS f ON (a,b) OR $(-\infty,\infty)$ OR (a,∞) OR $(-\infty,b)$

Ex: $f(x) = x^3 - 3x$; $[-1,2]$

Ex: $f(x) = \dfrac{1}{x^2+1}$

Ex: $f(x) = xe^{-x^2} \quad (-\infty, \infty)$