MATH 121

AN INTUITIVE APPROACH TO LIMITS

GOALS

- ☐ WRITE A CLEAR DEFINITION OF THE LIMIT OF A FUNCTION
- ☐ DEFINE ONE-SIDED LIMITS
  - ☐ USE ONE-SIDED LIMITS TO JUSTIFY WHETHER A LIMIT EXISTS OR NOT
- ☐ GIVEN THE GRAPH OF A FUNCTION, DETERMINE THE LIMIT OF f AS x APPROACHES A FINITE VALUE (FOR BOTH ONE- AND TWO-SIDED LIMITS)
- ☐ DEFINE VERTICAL ASYMPTOTES IN TERMS OF LIMITS
  - ☐ DEFINE $+\infty$, $-\infty$

A MOTIVATING QUESTION

Q: IS $0.\overline{9} = 1$? (OR IS IT "ALMOST" 1?)

RECALL: $0.\overline{9}$ MEANS , $0.\overline{12}$ MEANS

WE HAVE TO BE CLEARER ON WHAT WE MEAN HERE.

DEFINE A FUNCTION

$f(1) =$

$f(2) =$

$f(3) =$

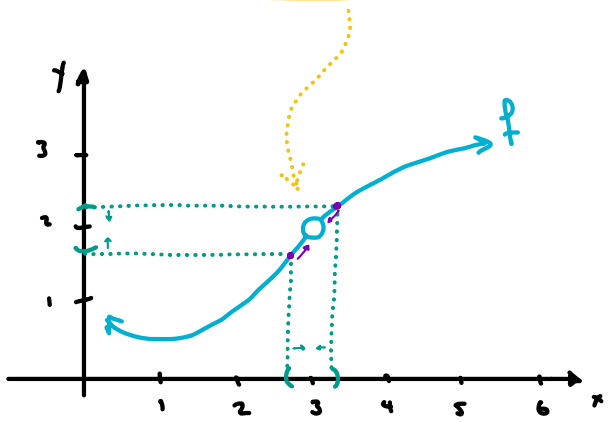
$\vdots$

$f(n) =$

WE CAN MAKE THE VALUE OF f

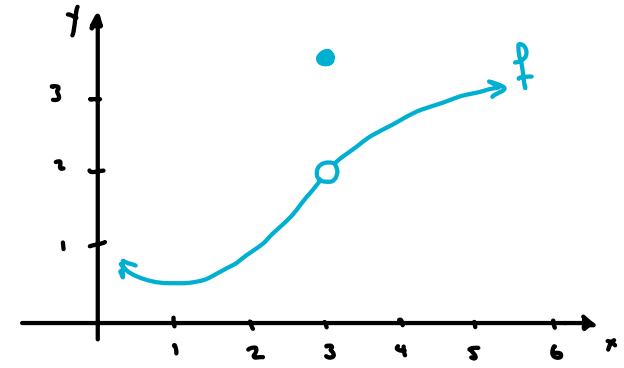
TRANSLATING THIS DEFINITION TO GRAPHS

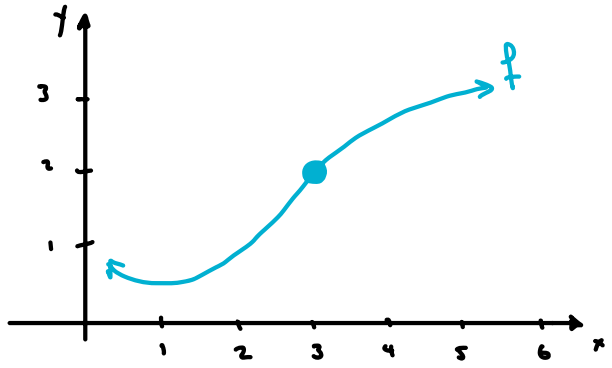
WHAT'S GOING ON AT $x = 3$ ON THIS GRAPH?



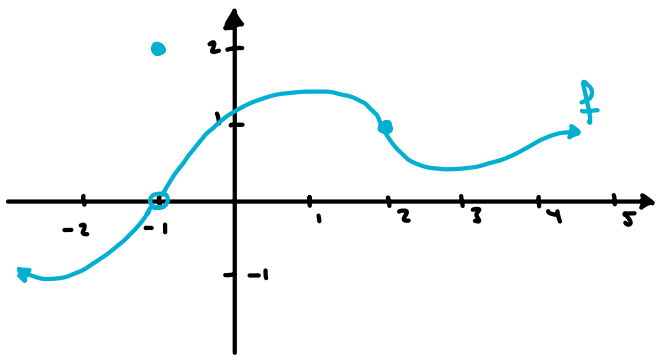
NOTE: WHAT $f(a)$ IS EQUAL TO AND WHAT $\lim_{x \to a} f(x)$ IS ARE <u>SEPARATE</u> QUESTIONS.

e.g.





EX DETERMINE $f(-1)$, $f(2)$, $\lim_{x \to -1} f(x)$, AND $\lim_{x \to 2} f(x)$ USING THE GRAPH



$f(-1) =$

$f(2) =$

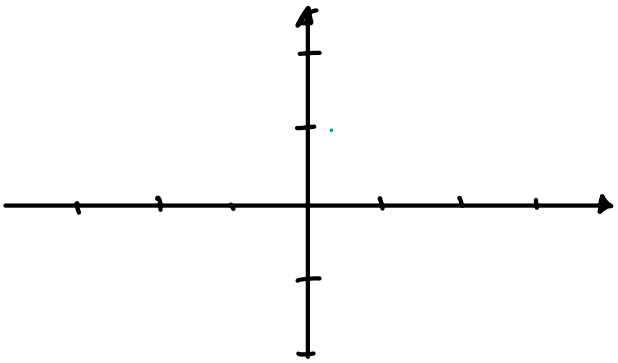
$\lim_{x \to -1} f(x) =$

$\lim_{x \to 2} f(x) =$

AN ODD EXAMPLE

SKETCH THE GRAPH OF $f(x) = \dfrac{|x|}{x}$

WHAT'S GOING ON AT $x = 0$?



ONE-SIDED LIMITS

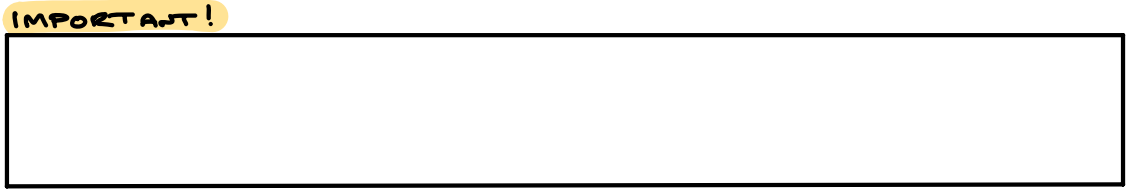
DEFINITION

(LEFT) $\lim\limits_{x \to a^-} f(x) = L$: WE CAN MAKE f AS CLOSE TO L AS WE LIKE

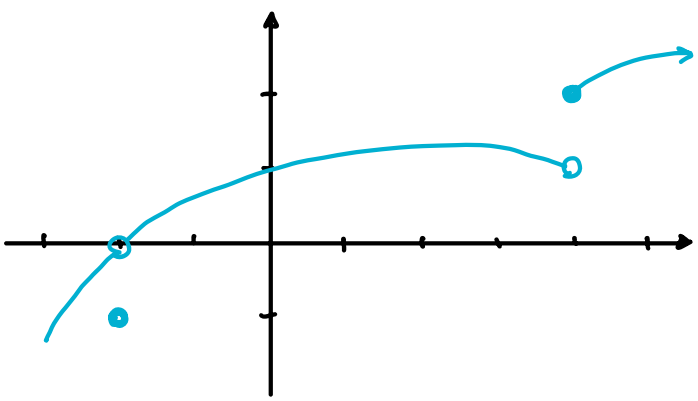
(RIGHT) $\lim\limits_{x \to a^+} f(x) = R$: WE CAN MAKE f AS CLOSE TO R AS WE LIKE BY

e.g. $\lim\limits_{x \to 0^-} \dfrac{|x|}{x} =$ WHILE $\lim\limits_{x \to 0^+} \dfrac{|x|}{x} =$

IMPORTANT!



EX DETERMINE $f(-2)$, $\lim\limits_{x \to -2} f(x)$, $f(4)$, $\lim\limits_{x \to 4} f(x)$. IF A LIMIT DNE, EXPLAIN.



$f(-2) =$

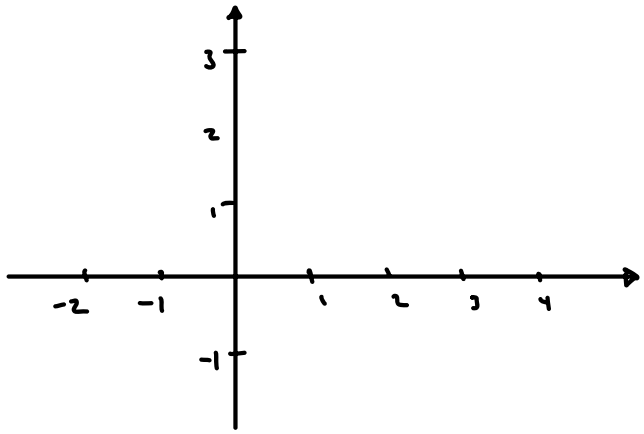
$\lim\limits_{x \to -2} f(x) =$

$f(4) =$

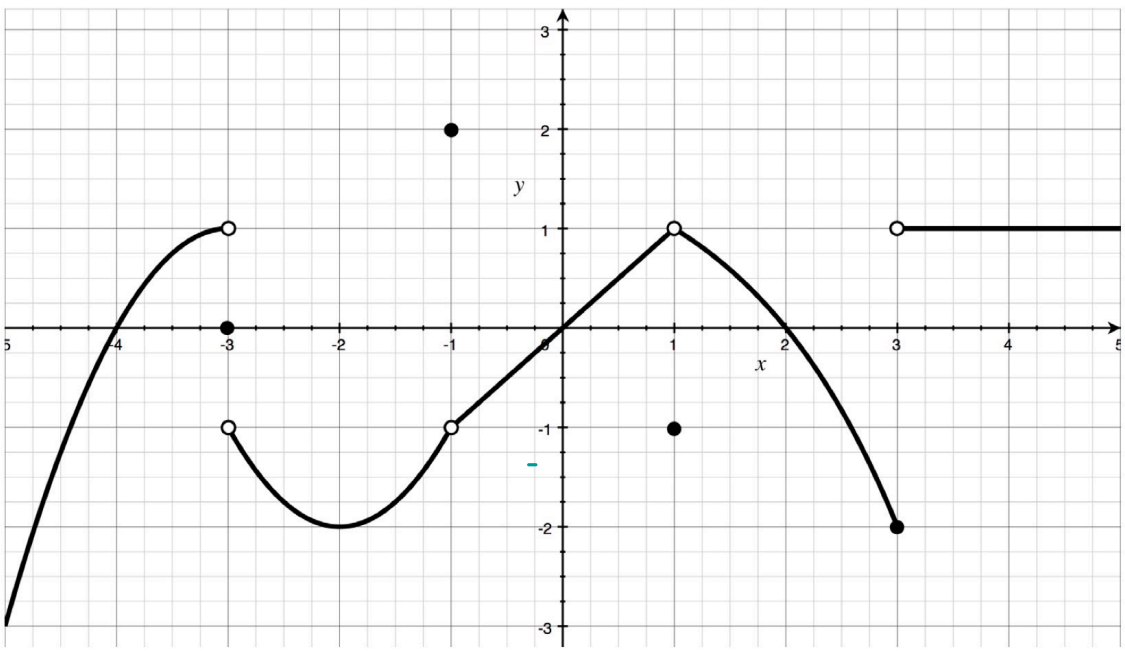
$\lim\limits_{x \to 4} f(x)$

EX SKETCH THE FOLLOWING PIECEWISE FUNCTION AND DETERMINE $f(0)$, $f(3)$, $\lim_{x\to 0} f(x)$, AND $\lim_{x\to 3} f(x)$.

$$f(x) = \begin{cases} x^2 - 1, & x \le 0 \\ 2, & 0 < x \le 3 \\ x - 1, & x > 3 \end{cases}$$



EX USE THE GRAPH TO DETERMINE THE ONE- AND TWO-SIDED LIMITS (IF THEY EXIST) AS WELL AS THE FUNCTION VALUES AT $x = -3, -1, 1, 3$



$\lim_{x\to 1^-} f(x) =$

$\lim_{x\to 1^+} f(x) =$

$\lim_{x\to 1} f(x) =$

$f(1) =$

$\lim_{x\to 3^-} f(x) =$

$\lim_{x\to 3^+} f(x) =$

$\lim_{x\to 3} f(x)$

$f(3) =$

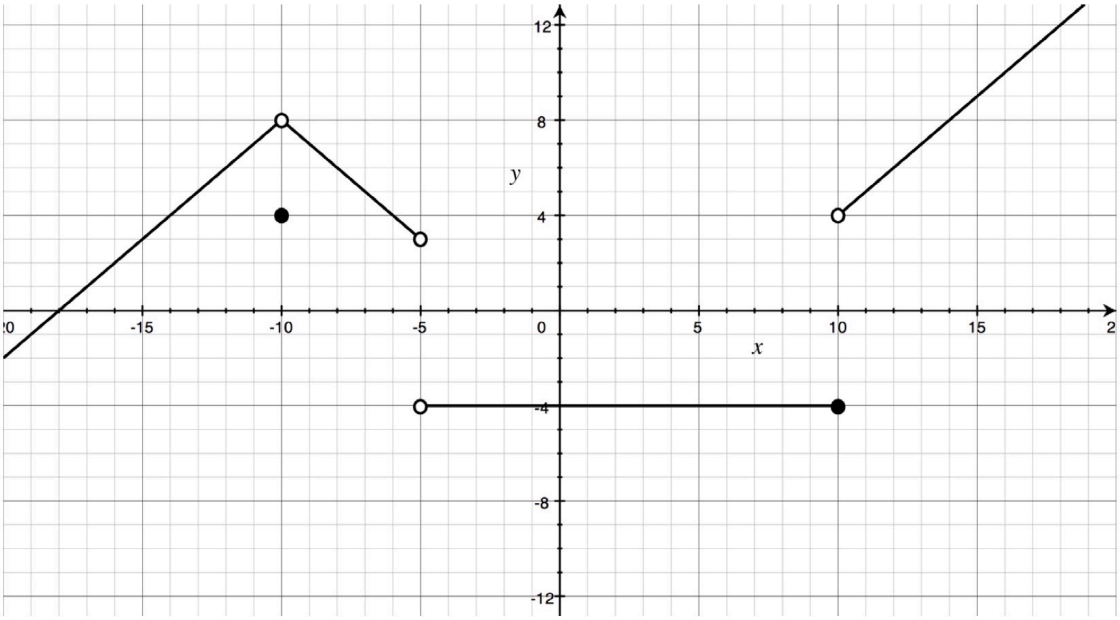
$\lim_{x\to -3^-} f(x) =$ $\lim_{x\to -3} f(x)$

$\lim_{x\to -3^+} f(x) =$ $f(-3) =$

$\lim_{x\to -1^-} f(x) =$ $\lim_{x\to -1} f(x) =$

$\lim_{x\to -1^+} f(x) =$ $f(-1) =$

EX USE THE GRAPH TO DETERMINE THE LIMITS AND FUNCTION VALUES AT $x = -10, -5, 10$ (IF A TWO-SIDE LIMIT DOESN'T EXIST, WRITE THE ONE-SIDED LIMITS)



$\lim_{x \to -10} f(x) =$

$f(-10) =$

$\lim_{x \to -5} f(x)$

$\lim_{x \to -5^+} f(x) =$

$\lim_{x \to -5^-} f(x) =$

$f(-5)$

$\lim_{x \to 10} f(x)$

$\lim_{x \to 10^+} f(x) =$

$\lim_{x \to 10} f(x) =$

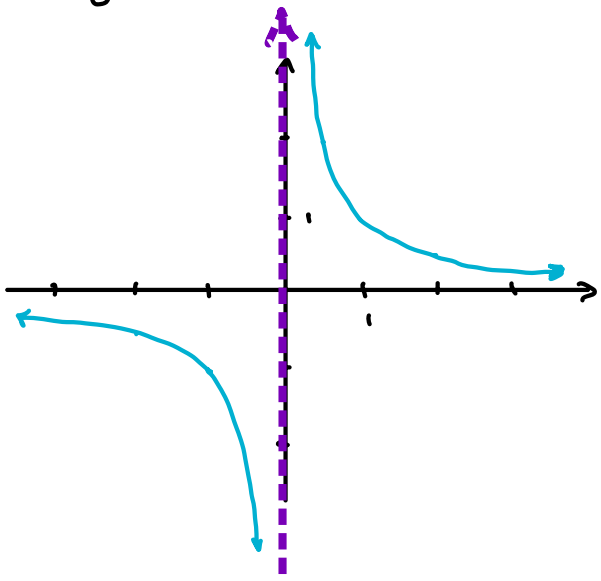
$f(10) =$

VERTICAL ASYMPTOTES

- **VISUAL AIDS** WE INCLUDE WITH A GRAPH TO

- VERT. ASYMPTOTES TELL US THAT $f(x)$

e.g. $f(x) = \frac{1}{x}$

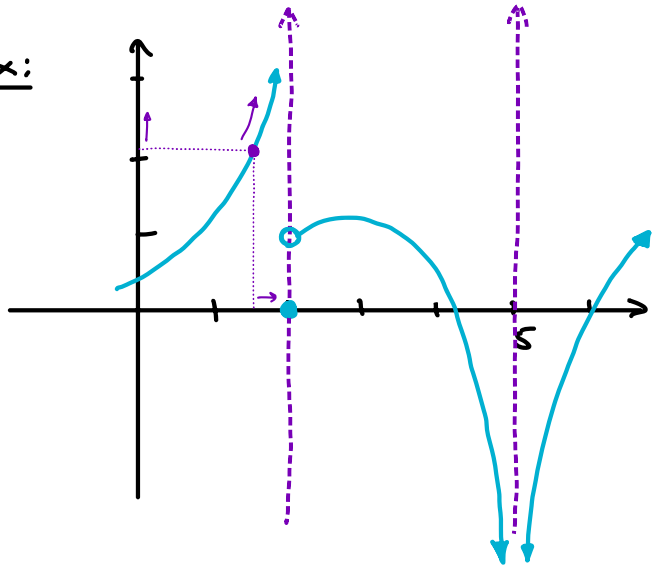


x	
$f(x)$	

WE WRITE...

IN GENERAL, WE HAVE AN VERTICAL ASYMPTOTE AT $x = a$ IF EITHER

EX:



$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2^+} f(x) =$

} $\lim_{x \to 2} f(x)$

$f(2) =$

$\lim_{x \to 5^-} f(x) =$

$\lim_{x \to 5^+} f(x) =$

} $\lim_{x \to 5} f(x)$

$f(5)$