Math 121

Exam 1

Monday: October 16th, 2023

Name:

Section:

Instructions

- This is a closed-book exam. You may *not* use any books or notes on this exam.
- For free response questions, you must show all work. Answers without proper justification will not receive full credit. Partial credit will be awarded for significant progress towards the correct answer. Cross off any work that you do not want graded.
- You have 50 minutes to complete this exam. When time is called, stop writing immediately and turn in your exam to the nearest proctor.
- You may not use any electronic devices except for the TI-30XIIS scientific calculator. Using any other device will be considered a violation of the university's academic integrity policy and, at the very least, will result in a grade of 0 for this exam.

Page	2	3	4	5	6	7
Points	20	20	18	16	14	12
Score						

Question (20 Points)

Consider the graph of f, shown below.

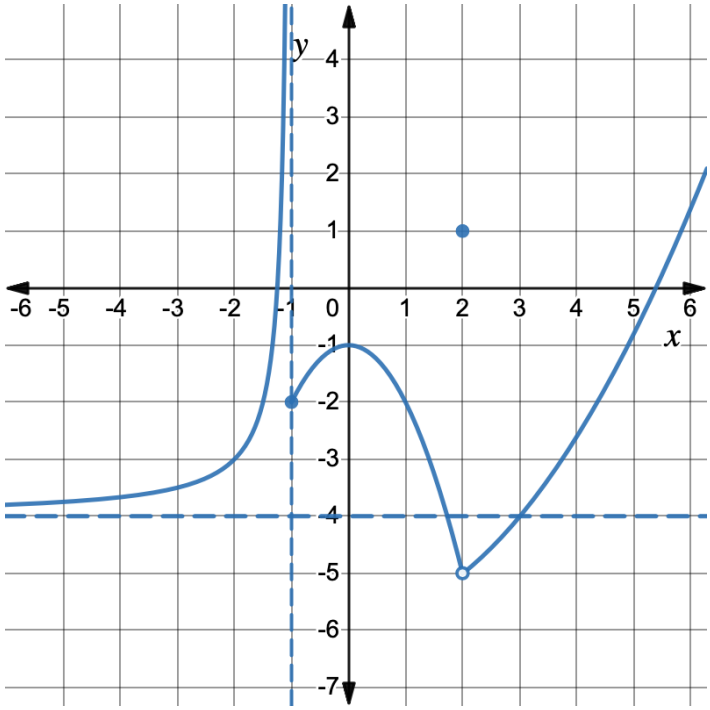

Figure 1: Piecewise Function

Find each of the following limits and function values. Find the limits. If a limit does not exist, write "+∞", "-∞", or "DNE" whichever is appropriate. If a function value is undefined, write UND.

	Answer		Answer
$\lim_{x\to -1^-} f(x) =$		$\lim_{x\to -1^+} f(x) =$	
$\lim_{x\to -1} f(x) =$		$\lim_{x\to 2^-} f(x) =$	
$\lim_{x\to 2^+} f(x) =$		$\lim_{x\to 2} f(x) =$	
$f(-1) =$		$f(2) =$	
$\lim_{x\to -\infty} f(x) =$		$\lim_{x\to +\infty} f(x) =$	

Solution

	Answer		Answer
$\lim_{x\to -1^-} f(x) =$	$+\infty$	$\lim_{x\to -1^+} f(x) =$	-2
$\lim_{x\to -1} f(x) =$	DNE	$\lim_{x\to 2^-} f(x) =$	-5
$\lim_{x\to 2^+} f(x) =$	-5	$\lim_{x\to 2} f(x) =$	-5
$f(-1) =$	-2	$f(2) =$	1
$\lim_{x\to -\infty} f(x) =$	-4	$\lim_{x\to +\infty} f(x) =$	$+\infty$

Question (7 points)

Evaluate the following limit.

$$\lim_{x\to -2} \frac{x^2+5x+6}{x^2+x-2}$$

Solution

$$\begin{aligned}
\lim_{x\to -2} \frac{x^2+5x+6}{x^2+x-2} &= \lim_{x\to -2} \frac{(x+2)(x+3)}{(x+2)(x-1)} \\
&= \lim_{x\to -2} \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-1)} \\
&= \lim_{x\to -2} \frac{(x+3)}{(x-1)} \\
&= \frac{(-2)+3}{((-2)-1)} \\
&= \boxed{-\frac{1}{3}}
\end{aligned}$$

Question (6 points)

Evaluate the following limit.

$$\lim_{x\to \pi} \left(\cos x + \ln \frac{x}{\pi}\right)$$

Solution

$$\lim_{x\to \pi} \left(\cos x + \ln \frac{x}{\pi}\right) = \cos \pi + \ln \frac{\pi}{\pi} = -1 + 0 = \boxed{-1}$$

Question (7 points)

Evaluate the following limit.

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$$

Solution

$$\begin{aligned}
\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} &= \lim_{x\to 9} \frac{\sqrt{x}-3}{(\sqrt{x}+3)(\sqrt{x}-3)} \\
&= \lim_{x\to 9} \frac{1}{\sqrt{x}+3} \\
&= \frac{1}{\sqrt{9}+3} \\
&= \boxed{\frac{1}{6}}
\end{aligned}$$

Question (10 Points)

Evaluate each of the following limits.

(*a*) $\lim_{x\to 2^-} \dfrac{4x-7}{x-2}$

(*b*) $\lim_{x\to 2^+} \dfrac{4x-7}{x-2}$

Solution

We can make a sign chart for $f(x) = \dfrac{4x-7}{x-2}$:

0	- - - -	UND	+ + + +
7/4	15/8	2	3

$$\lim_{x\to 2^-} \frac{4x-7}{x-2} = -\infty$$
$$\lim_{x\to 2^+} \frac{4x-7}{x-2} = +\infty$$

Question (8 Points)

Evaluate the following limits.

(*a*) $\lim_{x\to -\infty} \dfrac{\sqrt{4x^2+1}}{5x+2}$

Solution

$$\begin{aligned}
\lim_{x\to -\infty} \frac{\sqrt{4x^2+1}}{5x+2} &= \lim_{x\to -\infty} \frac{\sqrt{4x^2}}{5x} \\
&= \lim_{x\to -\infty} \frac{2|x|}{5x} \\
&= \lim_{x\to -\infty} \frac{2(-x)}{5x} \\
&= \lim_{x\to -\infty} -\frac{2}{5} \\
&= \boxed{-\frac{2}{5}}
\end{aligned}$$

(*b*) $\lim_{x\to +\infty} \dfrac{\sqrt{4x^2+1}}{5x+2}$

Solution

$$\begin{aligned}
\lim_{x\to+\infty} \frac{\sqrt{4x^2+1}}{5x+2} &= \lim_{x\to+\infty} \frac{\sqrt{4x^2}}{5x} \\
&= \lim_{x\to+\infty} \frac{2|x|}{5x} \\
&= \lim_{x\to+\infty} \frac{2(x)}{5x} \\
&= \lim_{x\to+\infty} \frac{2}{5} \\
&= \boxed{\frac{2}{5}}
\end{aligned}$$

Question (6 Points)

Evaluate the following limits.

(*a*) $\lim_{x\to-\infty} \dfrac{5x^4+3x-2}{2x^3+x^2+x+1}$

Solution

$$\begin{aligned}\lim_{x\to-\infty} \frac{5x^4+3x-2}{2x^3+x^2+x+1} &= \lim_{x\to-\infty} \frac{5x^4}{2x^3}\\ &= \lim_{x\to-\infty} \frac{5}{2}x\\ &= \boxed{-\infty}\end{aligned}$$

Question 8 (10 Points)

Evaluate the following limits.

(a) $\lim_{x\to-\infty} \dfrac{2x+4}{3x^3-x+8}$

Solution

$$\begin{aligned}\lim_{x\to-\infty} \frac{2x+4}{3x^3-x+8} &= \lim_{x\to-\infty} \frac{2x}{3x^3}\\ &= \lim_{x\to-\infty} \frac{2}{3x^2}\\ &= \boxed{0}\end{aligned}$$

(b) $\lim_{x\to\infty} \dfrac{8x^3+4x^2+x-5}{4x^3+x^2-7x+1}$

Solution

$$\begin{aligned}\lim_{x\to\infty} \frac{8x^3+4x^2+x-5}{4x^3+x^2-7x+1} &= \lim_{x\to\infty} \frac{8x^3}{4x^3}\\ &= \lim_{x\to\infty} \frac{8}{4}\\ &= \boxed{2}\end{aligned}$$

Question (8 Points)

Consider the piecewise function f:

$$f(x) = \begin{cases} \dfrac{\sin 2x}{x}, & x < 0 \\ ax + b, & 0 \le x \le 2 \\ x^2 - 10, & x > 2 \end{cases}$$

Determine the values of a and b that make f continuous everywhere.

Solution

$$\begin{aligned}
\lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-} \frac{\sin 2x}{x} \\
&= 2 \\
\lim_{x\to 0^+} f(x) &= \lim_{x\to 0^-} ax + b \\
&= b \\
\lim_{x\to 0^-} f(x) &= \lim_{x\to 0^+} f(x) \\
b &= 2 \\
\lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} ax + b \\
&= a(2) + 2 \\
&= 2a + 2 \\
\lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} x^2 - 10 \\
&= (2)^2 - 10 \\
&= -6 \\
\lim_{x\to 2^-} f(x) &= \lim_{x\to 2^+} f(x) \\
2a + 2 &= -8 \\
a &= -4
\end{aligned}$$

Question (6 Points)

Evaluate the following limit.

$$\lim_{x\to 0} \frac{\tan 3x}{\sin 5x}$$

Solution

$$\begin{aligned}
\lim_{x\to 0} \frac{\tan 3x}{\sin 5x} &= \lim_{x\to 0} \frac{\sin 3x}{\cos 3x \sin 5x} \cdot \frac{x}{x} \cdot \frac{3}{3} \cdot \frac{5}{5} \\
&= \lim_{x\to 0} \frac{3}{5\cos 3x} \cdot \frac{\sin 3x}{3x} \cdot \frac{5x}{\sin 5x} \\
&= \frac{3}{5(1)} \cdot 1 \cdot 1 \\
&= \boxed{\frac{3}{5}}
\end{aligned}$$

Question (6 Points)

Evaluate the following limit.

$$\lim_{x \to -\infty} \frac{3e^x + 4}{5e^x + 9}$$

Solution

$$\lim_{x \to -\infty} \frac{3e^x + 4}{5e^x + 9} = \frac{3(0) + 4}{5(0) + 9}$$

$$\boxed{\frac{4}{9}}$$

Question (6 Points)

Show that $f(x) = x^3 - x^2 + 3x - 2$ has at least one x-intercept in the interval $(0, 1)$.

Solution

The function f is continuous for all real numbers. Since $f(0) = -2 < 0$ and $f(1) = 1 > 0$, the intermediate value theorem tells us that for some c in $(0, 1)$, $f(c) = 0$.